SUSPENSION BRIDGES

Roberto Crocetti
How does the tension in the cable vary with increasing sag ”f”?
Design of cable

- Span: L
- Total weight: q
Funicular of forces

• Divide the uniformly distributed load into a sufficient number of segments
• Assign a force to each segment

\[ F_i = q \Delta x \]
Funicular of forces

• Find vertical reactions
Funicular of forces

• Find vertical reactions
• Choose a point “O”
• The maximum force in the cable can be obtained by measuring the segment O-I:

\[ F_{AB} \approx 8.5 \cdot F_i \]

Or:

\[ F_{AB} = \frac{R_A}{\sin \alpha} \]

For a given cable cross section and strength, the position of “O” can be determined graphically.
The sag “f” is too big!

We want f=L/10

How do we achieve a smaller sag?
Move the origin from “O” to “O’”

Note that doubling the distance between $O$ and the vertical line the sag $f$ is reduced by one half.

However, the force in the cable has significantly increased!
Cable length

\[ ds^2 = (dx)^2 + dy^2 \Rightarrow \]

\[ ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \]

\[ u_x = \frac{\int_0^L x^2 \ dx}{L^2} \Rightarrow \frac{du_x}{dx} = \frac{8}{L^2} x \]

\[ L^2 = \int_0^L \sqrt{1 + \left( \frac{8}{L^2} x \right)^2} \ dx \approx L \left( 1 + \frac{8}{3} \left( \frac{f}{L} \right)^2 \right) \]
Example
Example

\[ f = 10 \text{ m} \Rightarrow \frac{f}{L} = 0.1 \]

\[ L' = 100 \left( 1 + \frac{B}{3} (0.1)^2 \right) = 102.7 \text{ m} \quad \text{(ONLY } 2.7\% \text{ LONGER THAN } "L"
) \]
Some important properties of cables
Cable subjected to uniformly distributed load

\[ M_{g,beam} = \frac{g \cdot l}{2} \cdot x - \frac{g}{2} \cdot x^2 \]

\[ y = \frac{M_{g,beam}}{H_g} \]

\[ H_g = \frac{g \cdot l^2}{8 \cdot f} \]
Deflection: larger self weight induce smaller deflection due to concentrated load!
Influence of the bridge self-weight on the deflection caused by a concentrated load – *a graphic-statics approach*
Cable subjected only to concentrated load $P$

- Consider a cable with sag $f = L/5$ subjected to a concentrated load $P$
Consider a cable with sag $f=L/5$ subjected to a concentrated load $P$. 

Cable subjected only to concentrated load $P$.
Self weight + concentrated load
Comparison
Which is the position of the concentrated load that produces the maximum deflection of the cable?
Deformation of the cable for different positions of "Q" (analytically derived curves)

Position of "Q" for maximum deflection (1/5 of the span)

Ratio between point load and self-weight (unif. distributed)

\[ \frac{Q}{g\ell} \]

\[ \eta_{oil} \]
How can we increase the stiffness of a cable?

*in other words*

how can we reduce the vertical deflection of a cable caused by a point load?
Methods to reduce the deformation of cable structure

- **Increase self-weight**

- **Pre-stress the hangers**

- **Cable with a certain bending stiffness (beam/cable)**

- **Use a stiffer stiffening girder which can redistribute the concentrated load**
What is the effect of the self weight?

Larger self-weight induce larger tension “T” in the cable. “T” can be regarded as a pre-stressing force.

The higher the pre-stress force, the stiffer is the cable (compare to a guitar string)
Light deck (small pre-stress in the cable)

Large deformations to concentrated load

Heavy deck (large pre-stress in the cable)

Small deformations to concentrated load
Increase the bending stiffness of the cable

Braced chain suspension bridge
Increase the bending stiffness of the cable
Increase the stiffness of the deck (in other words: use a stiff stiffening girder)
Increase the stiffness of the deck (in other words: use a stiff stiffening girder)

If the deck is stiff enough, the shape of the cable remains a parabola under loading. This means that the cable must be subjected to uniformly distributed load!
Suspension bridges with stiffening girder

• The stiffening girder transforms the concentrated load into a distributed set of equal vertical pulls that are compatible with the shape of the cable

• All the differences between the actual loading and the loading that corresponds to the shape of the cable are absorbed by the beam
• Show example
Bending moment in the stiffening

Notice that most of the self weight is taken by the cable!
Structural analysis of suspension brides
Real situation: Interaction between cable and deck

Sketch: B. Åkesson
Melan’s theory

• Hooke’s law applies for all the components

• Hangers closely placed (the suspending force can be considered as uniformly distributed)

• Stretching of the cable negligible

• Elongation of the hangers and misalignment negligible
Simplification

Sketch: B. Åkesson
Melan’s differential equation

Cable force equilibrium:

1. \( (H_g + H_Q) \cdot (y + \eta) = M_{\text{beam}} \)

Derivate two times:

2. \( (H_g + H_Q) \cdot (y'' + \eta'') = -s \)

Remember that:

\[
\frac{\partial^2 M}{\partial x^2} = -q
\]
Melan’s differential equation

Differential equation for the girder:

3. \( E \cdot I \cdot \eta^{IV} = g + q - s \)

Put 2. in 3.:

4. \( E \cdot I \cdot \eta^{IV} = g + q + (H_g + H_Q) \cdot \left(y'' + \eta''\right) \)
Melan’s differential equation

Equation 4. can be rewritten in the following way:

5. \( E \cdot I \cdot \eta^{IV} = g + q + H_g \cdot y'' + H_g \cdot \eta'' + H_q \cdot y'' + H_q \cdot \eta'' \)
Melan’s differential equation

It can be assumed that the cable carries the self-weight alone (i.e. the girder is not contributing to carry the self weight)

6. \((H_g) \cdot (y) = M_g\) \(\Rightarrow (H_g) \cdot (y'') = -g\)

Put 6. in 5.:

7. \(E \cdot I \cdot \eta^{IV} = g + q - g + H_g \cdot \eta'' + H_q \cdot y'' + H_q \cdot \eta'' \Rightarrow\)

\[\Rightarrow E \cdot I \cdot \eta^{IV} = q + (H_g + H_q) \cdot \eta'' + H_q \cdot y''\]
Or integrating Eq. 7 two times:

\[ M = M_q + \left( H_g + H_q \right) \cdot \eta + H_q \cdot y \]

\( M_q \): Bending moment due to “q” acting on a simply supported beam

\( M \): Bending moment in the girder
Note that for slender girders (i.e. $EI/L$ small) the equation of the cable alone leads to similar results as the Melan’s equation.
Suspension bridge components
Some main suspension bridges

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Country</th>
<th>Span</th>
<th>Opening Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><em>Messina Bridge</em></td>
<td>Italy</td>
<td>3,300 m</td>
<td>2020</td>
</tr>
<tr>
<td>2</td>
<td>Akashi Kaikyō Bridge</td>
<td>Japan</td>
<td>1,991 m</td>
<td>1998</td>
</tr>
<tr>
<td>3</td>
<td>Xihoumen Bridge</td>
<td>China</td>
<td>1,650 m</td>
<td>2009</td>
</tr>
<tr>
<td>4</td>
<td>Great Belt Bridge</td>
<td>Denmark</td>
<td>1,624 m</td>
<td>1998</td>
</tr>
<tr>
<td>5</td>
<td><em>Izmıt Bay Crossing</em></td>
<td>Turkey</td>
<td>1,550 m</td>
<td>2015</td>
</tr>
<tr>
<td>6</td>
<td><em>Yi Sun-sin bridge</em></td>
<td>South Korea</td>
<td>1,545 m</td>
<td>2012</td>
</tr>
<tr>
<td>7</td>
<td>Runyang Bridge</td>
<td>China</td>
<td>1,490 m</td>
<td>2005</td>
</tr>
<tr>
<td>8</td>
<td><em>Nanjing Fourth Yangtze Bridge</em></td>
<td>China</td>
<td>1,418 m</td>
<td>2011</td>
</tr>
<tr>
<td>9</td>
<td>Humber Bridge</td>
<td>England, UK</td>
<td>1,410 m</td>
<td>1981</td>
</tr>
<tr>
<td>10</td>
<td>Jiangyin Suspension Bridge</td>
<td>China</td>
<td>1,385 m</td>
<td>1999</td>
</tr>
<tr>
<td>11</td>
<td><em>Tsing Ma Bridge</em></td>
<td>Hong Kong</td>
<td>1,377 m</td>
<td>1997</td>
</tr>
</tbody>
</table>
Akashi Kaikio Bridge, Japan 1991m

Cable diameter ca 1,2m
The Älvsborg Bridge, 1996 (Sweden)

Höga Kusten, 1998 (Sweden)

Golden Gate Bridge, 1937 (USA)

Humber Bridge, 1981 (England)

The Great Belt, 1981 (Denmark)

Akashi-Kaikyo, 1998 (Japan)

Bridge over the Messina Strait (Italian)
Will open for traffic in June 2013
Hardanger Bridge

1310 meter

Hardangerbrua
Types of suspension bridges

Single-Span

Three-Span

Multi-Span
Types of stiffening girders

- Two-hinged Stiffening Girder
- Continuous Stiffening Girder
Types of suspenders/hangers
Example: Severn bridge (UK)

- the bridge becomes more rigid (» 25%), due to the truss behaviour
- reduced tendency to oscillate (flutter).
- However, the constantly changing forces in the hangers can create fatigue problems
Type of anchoring

Externally-anchored Type

Self-anchored Type
Sag-to-length ratio

Classical Long Span Concept (3 span, 2 hinge)

East Bridge (Continuous Girder, Central Tie)
Great Belt suspension bridge
Great Belt Bridge, Overview

Total length: 6.8 km
Let us study the different parts/components of a suspension bridge
Towers/Pylons

- Rigid towers for *multispan suspension* bridges to provide enough stiffness to the bridge.

- Flexible towers are commonly used in *long-span* suspension bridges.

- Rocker towers occasionally for relatively *short-span* suspension bridges.
Some common tower types

<table>
<thead>
<tr>
<th>Shape</th>
<th>Truss</th>
<th>Portal</th>
<th>Combined Truss and Portal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>![Truss Diagram]</td>
<td>![Portal Diagram]</td>
<td>![Combined Truss and Portal Diagram]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bridge</th>
<th>Akashi Kaikyo Forth Road</th>
<th>Great Belt East Humber</th>
<th>Golden Gate Second Tacoma Narrows</th>
</tr>
</thead>
</table>
The pylons, Hardanger Bridge
Great belt Bridge, Pylon

- Height above sea level: 254 m
- Legs at top: 6.5 m x 7.5 m
- Legs at base: 14 m x 15 m
- Caisson: 78 m x 35 m
- Wall thickness of legs: 1.7 m
- Concrete per pylon: 51,250 m³
Common stiffening girder

I-girder
(Bronx-Whitestone Bridge)

Truss Girder
(Mackinac Straits Bridge)

Box Girder
(Humber Bridge)
Great belt Bridge, Girder

- Total length: 2,694 m
- Length of sections: 48 m
The stiffening girder, Hardanger Bridge
Messina strait bridge, closed box girder
Cables

• A cable is a highly flexible member

• A cable transmits primarily axial forces

• A cable can be made of:
  • a bundle of steel wires
  • A bundle of strands
  • A bundle of several cables
Wires

- Are produced from high-strength steel bars by rolling or cold drawing (the initial area is reduced)

- The cold-forming process results in
  - an increase in the tensile and yield stress and
  - a decrease in the ductility of the

Diametre 1-7 mm
Wires

• The optimum wire diameter is 5,0-5,5mm

• A larger diameter makes the wire too stiff

• A smaller diameter requires more wires and more labour.

• The wire material has an ultimate strength up to 1600 - 1800N/mm²
Strand

- is produced from a series of wires that are wound together in a helical, parallel or Z-lock fashion
Cables/ropes

Parallel wire cables are composed of a series of parallel wires.

Strand cables are composed of parallel or helically combined strands.

Locked-coil cables (which were invented for better corrosion protection). These cables are less flexible than the other types.
## Types of cables

<table>
<thead>
<tr>
<th>Name</th>
<th>Shape of section</th>
<th>Structure</th>
<th>Bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel Wire Strand</td>
<td><img src="image1" alt="Hexagonal Wires" /></td>
<td>Wires are hexagonally bundled in parallel.</td>
<td>Brooklyn, Humber, Great Belt East, Akashi Kaikyo</td>
</tr>
<tr>
<td>Strand Rope</td>
<td><img src="image2" alt="Round Wires" /></td>
<td>Six strands made of several wires are closed around a core strand.</td>
<td>St. Johns</td>
</tr>
<tr>
<td>Spiral Rope</td>
<td><img src="image3" alt="Spiral Wires" /></td>
<td>Wires are stranded in several layers mainly in opposite lay directions.</td>
<td>Little Belt, Tancarville, Wakato</td>
</tr>
<tr>
<td>Locked Coil Rope</td>
<td><img src="image4" alt="Coiled Wires" /></td>
<td>Deformed wires are used for the outside layers of Spiral Rope.</td>
<td>Kvalsund, Emmerich, Ålvsborg, New Köln Rodenkirchen</td>
</tr>
</tbody>
</table>
Suspension cables of the Hardager bridge - Norway

Kabelbunten med 528 galvaniserte tråder å 5,3 mm. I alt 10.032 tråder. Bruddlast for kabelen 342.030 kN

Kabelen er spunnet av galvaniserte ståltråder med diameter 5,3 mm. Kabeltråden har en strekkfasthet på 1570 MPa.
Hanger cable of the Hardager bridge - Norway

Det er totalt 130 hengestenger. Lengden varierer fra 2,04 m til 127,6 m. De fem korteste er uten kabel, og produsert i ett stykke støpestål. Hengestangskabelen er en «lukket kabel» som består av sju lag med tråder hvor de tre ytterste er Z-tråder. Hengestang og kabel/stålkasse forbindes med bolter Ø160 mm.
KABEL OG HENGESTENGERT

Brua bæres av kabel og hengestenger.

Kabelen er bygget opp av galvaniserte stålstråder. Den bygges ved at tråden trekkes på et spinnehjul som dras på en taubane. På hver tur dras to eller fire tråder. Trådene samles i 19 bunter med 528 tråder i hver bunt. Så kompakteres alle buntene til et sirkulært tverrsnitt ved hjelp av en hydraulisk presse.

Deretter monteres hengestangsfestene. Etter det vikles kabelen med en myk galvanisert tråd som beskytter kabelen og gjør at den beholder formen. Utenpå vikletråden legges armert tape som beskytter mot fukt og regn. Selv om trådene i kabelen ligger tett er det fremdeles ca 20% luft inne i kabelen. Kablene avfuktet ved at tørr luft blåses gjennom kablene.

Så monteres hengestengene som består av en kabel med et støpestålsbode i hver ende. Hengestangskabelen er forankret i støpestålsboden ved hjelp av en utstøpt konusformet zinkblokk.
Mechanical properties of cables

• the tensile strength of the wires is high (it is also normally inversely proportional to the wire diameter). Normally \( f_u = 1500-1800 \text{ N/mm}^2 \).

• The \( \sigma - \varepsilon \) diagram for this steel has no yield plateau and so the yield strength is conventionally defined as the stress at which the plastic deformation is 0.2%. \( f_y = 80 \text{ to } 90\% \text{ of } f_u \)

• The modulus of elasticity of cables is generally smaller than that of the steel material (wire) of which they are composed
Stiffness of cables

- For parallel wire cables $E = 200 \text{ N/mm}^2$
- For locked-coil cables $E = 160 \text{ N/mm}^2$
- For strand cables $E = 150 \text{ N/mm}^2$
Design values for cables (check producers web-page)

The tensile resistance of a cable is:

$$F_u = k_s A_m \cdot f_u$$

Am is the “metallic area” of a cable:

$$A_m = f \frac{\pi d^2}{4}$$

d = cable diameter

$$f = 0.55 - 0.86$$

The design tensile resistance is

$$F_{Rd} = \frac{F_u}{\gamma_M} \quad (\gamma_M = 2.0 \text{ for cables})$$

$$k_s = 0.76 - 1.0$$
Main Cables Great Belt bridge - DK

BEFORE COMPACTION

AFTER WRAPPING

strand
Connections cable-hanger and girder-hanger

![Diagram showing connections](image)

- **Bearing connection**
- **Pin plate connection**
Cable and hangers, Hardanger Bridge
Attaching the cables – socketed fitting

The end of the cable is broomed into individual strands.
Attaching the cables – socketed fitting

The broomed end of the cable is then inserted into a conical steel basket.

Molten zinc is poured into the basket to embed the wires and make the attachment.
Connections

- Saddling
- Anchorage
- Coupling
- Anchorage
A preliminary sketch of details for a relatively small cable structure (cable, mast and foundation)
Saddles
The tower saddle, Hardanger Bridge
Cable anchorage – Severn bridge (UK)
The anchor block, Hardanger Bridge

19 strands anchored to the anchoring devise
The anchor block, Hardanger Bridge
Great belt Bridge, Girder Aerodynamics
Wind tunnel studies

Stoncutters Bridge - Wind Tunnel Testing
## KOSTNADAR

<table>
<thead>
<tr>
<th>Hovedplassering</th>
<th>Beløp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tårn</td>
<td>260 mill kr</td>
</tr>
<tr>
<td>Viadukter</td>
<td>50 mill kr</td>
</tr>
<tr>
<td>Forankringar</td>
<td>150 mill kr</td>
</tr>
<tr>
<td>Kablar og hengstenger</td>
<td>420 mill kr</td>
</tr>
<tr>
<td>Stålkasse</td>
<td>450 mill kr</td>
</tr>
<tr>
<td>Membran og asfalt</td>
<td>20 mill kr</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>1350 mill kr</strong></td>
</tr>
<tr>
<td>Tiltak av forankringar</td>
<td>400 mill kr</td>
</tr>
<tr>
<td>Byggherreforankringar</td>
<td>300 mill kr</td>
</tr>
<tr>
<td><strong>Styringsramme (totalsum)</strong></td>
<td><strong>2300 mill kr</strong></td>
</tr>
</tbody>
</table>

## FRAMDRIFT OG BYGGETID

- Hardangerburuprosjektet godkjent februar 2006, skal stå ferdig våren 2013
- Prosjektering og planlegging, 2 år (mars 2006 - mai 2008)
- Bygging av tunnel og bilferderveg, byggeperiode fra februar 2009 til desember 2010, ferdig våren 2013
- Bygging av bygning, kring 3,5 år, byggestart august 2009, ferdig våren 2013
- Tårn og forankringar, 1,5 år
- Kablar og hengstenger, kring 1 år
- Montering av stålkasse og komplettassembling, kring 1 år
- Stålkasse og en del av kabel blir produsert i verkstad parallel med tårn og forankringar, produkjonstid kring 2 år