Dynamic performance of pedestrian bridges

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Dynamic performance of pedestrian bridges
- Footfall induced vibrations

Dynamisk analys av GC-broar med hänsyn till gångtrafik

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Abstract

The recent developments in materials, construction methods and design procedures have allowed engineers to design lighter, more material efficient and longer bridges. Due to requirements ensuring the comfort of a pedestrian the vibrations induced by walking often becomes the limiting factor of a design. The Millennium bridge in London was famously closed down immediately after opening due to large lateral vibration induced by the crowd, it was evident that the current knowledge did not capture the full scope of the phenomena. To quantify the human footfall and its effect on a footbridge during crowd loading have been the aim of many authors, yet no clear guidelines on the subject exists today. Instead the problem is often left to the designer.

This thesis aims to present and clarify some of the proposed design methods aimed to capture the effects of footfall induced vibrations. In order to understand how these methods are derived the necessary theory is presented allowing for a deeper understanding of dynamic loads as well as structural dynamics in general. The dynamic response induced by a load model is commonly acquired using a FE-software, therefore, it is also within the scope of this project to study some of the problems commonly encountered when simulating dynamic loads. The use of generalized SDOF-systems is also studied for its lower computational cost and applicability to hand calculations. This is presented to the reader as case studies where two footbridges are evaluated and the results compared.

In this project the large variation in different load models has been shown and how the response depends on the specific model used. The large variation in load parameters are due to the uncertainties associated with pedestrian behaviour resulting in often conservative models. In order to capture the behaviour more accurately and establish a more efficient design method it seems that further studies on the human footfall and the pedestrian behaviour in crowds is required. It has also been shown through comparative case studies that the use of generalized SDOF-systems captures the response fairly accurately. A FE-software is still used in order to acquire the necessary modal data, however, simplified assumptions regarding the modal shapes can be made in order to allow for hand calculations. These are useful for either verifying FE-results or as a preliminary design tool.

Although the established guidelines provide only limited support, leaving often the problem to the designer, there are helpful publications on the subject of footfall induced vibrations. However, it requires some knowledge of fundamental structural dynamics by the designer in order to efficiently assess the outcome of various parameters and assumptions. Unfortunately there is still a lot of research left in order to fully understand the dynamic behaviour and properties of structures as well as the human footfall. Studying the dynamic properties of completed structures through testing and improving on current methods based on those findings is necessary in order to allow for more efficient design methods in the future.
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Chapter 1

Introduction

1.1 Background

In recent years the dynamic properties have become one of the major limitations when designing a bridge for pedestrian use. Due to developments in materials, construction methods and design procedures structures can be made lighter and more material efficient. This has given rise to dynamic problems in terms of footfall induced vibrations in the serviceability limit state. Since this a fairly recent issue, guidelines on this subject are, at best, limited [10][11][12].

Recently, a few documents have been published within which attempts have been made to provide a complete design method regarding footfall induced vibrations on a footbridge [3][4][5]. However, some of these methods are rather difficult to understand or even apply due to their basis on equations not properly presented through derivations. Furthermore, these methods often rely on FE-softwares to compute the response through time-stepping procedures. Not only is this method time consuming, it also limits the possibility to verify computed results or even understand the significance of different parameters governing the dynamic response.

In a document published by the Concrete Centre, M.R. Willford and P. Young describe a more analytical approach to structural dynamics which makes use of generalized SDOF - systems and thereby significantly simplifies the problem of calculating the response [2]. Using this method, even complex structures can be analysed primarily by hand which both increases the understanding of structural dynamics and provides insight in the parameters governing the dynamic response. However, this document does not provide a complete design method concerning different loading situations in terms of crowd loading or even the possibility to account for horizontal vibrations. Furthermore, it only covers the use of single point loads. The absence of derivation for the presented equations complicates the application of other types of loads such as distributed loads. These factors limit the applicability of the method as a practical design tool.

The pedestrian loads are commonly treated in a deterministic manner where the load is simulated using Fourier series where the parameters are determined through measurements of the human footfall. However, these load model parameters usually differ substantially depending on the method used.
1.2 Objectives

The objective of this report is to study the currently available methods and the underlying theory in order to determine each method’s applicability as a design tool. The aim is to both introduce the reader to these methods as well as the theory on which these methods are based on in order to increase the understanding of structural dynamics and its practical applications.

The main part of this report will be focused on explaining the methods and models presented by Sétra and Willford & Young. The aim is to clarify the equations presented by Willford & Young with complete derivations and examine a more general form of this method. Its accuracy will be determined through comparisons with dynamic FE-analysis. Furthermore, the various problems that arise when modelling a footbridge in a FE-software are described and how these problems influence the response in order to increase the readers understanding of the finite element method.

The two methods by Sétra and Willford & Young are based on different pedestrian load parameters, a variety of load models have been proposed by many different authors as well. It is therefore also a part of this report to compare these models and how they influence the dynamic response.

Finally, comparative studies will be performed on two footbridges in order to further illustrate the different methods and compare the influence of various parameters in the pedestrian load model.
1.3 Cases

In order to compare and clarify these methods to the reader two case studies are performed. These footbridges are designed by WSP department of bridge engineering. Complete models in the FE-software BRIGADE/plus and calculations were provided by WSP for this report.

1.3.1 Bro 4-883-1 Munktell

The first case is a pedestrian beam bridge designed by WSP according to the Swedish guidelines TRVK Bro 11 and TK Geo 11. The effective width of the footbridge is 4.5 m, the lengths of the outer spans are 16.5 m and the central span is 32 m. The deck connects to the supports through arches and columns, see figure 1.1 and 1.2. A summary of the parts used in the model is presented in table 1.1.

![Munktell - Overview 1](image1)

![Munktell - Overview 2](image2)

Table 1.1: Brief summary of parts in case Munktell.

<table>
<thead>
<tr>
<th>Part</th>
<th>Material</th>
<th>Element</th>
<th>Profile</th>
<th>Dimensions [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck</td>
<td>C35/45</td>
<td>Shell</td>
<td>Plate</td>
<td>4.9 · 65.5 ( t = 0.2 )</td>
</tr>
<tr>
<td>Edge Beam</td>
<td>S355</td>
<td>Beam</td>
<td>Rectangular, Hollow</td>
<td>0.4 · 0.7 ( t = 0.025 )</td>
</tr>
<tr>
<td>Column</td>
<td>S355</td>
<td>Beam</td>
<td>Rectangular, Hollow</td>
<td>0.4 · 0.7 ( t = 0.025 )</td>
</tr>
<tr>
<td>Arch</td>
<td>S355</td>
<td>Beam</td>
<td>Rectangular, Hollow</td>
<td>0.36 · 0.5 ( t = 0.025 )</td>
</tr>
<tr>
<td>Cross beam</td>
<td>S355</td>
<td>Beam</td>
<td>HEB240</td>
<td>-</td>
</tr>
<tr>
<td>Support</td>
<td>C35/45</td>
<td>Solid</td>
<td>Rectangular, Solid</td>
<td>2.5 · 1</td>
</tr>
</tbody>
</table>
The material parameters used are presented in table 1.2.

Table 1.2: Material properties for case Munktell.

<table>
<thead>
<tr>
<th>Material</th>
<th>$f_{ck}$ [MPa]</th>
<th>$f_{cd}$ [MPa]</th>
<th>$f_{ck}$ [MPa]</th>
<th>$f_{ctd}$ [MPa]</th>
<th>$E_{cd}$ [GPa]</th>
<th>$\gamma$ [kN/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>35</td>
<td>23.33</td>
<td>2.2</td>
<td>1.47</td>
<td>28.33</td>
<td>25</td>
</tr>
<tr>
<td>Rebar</td>
<td>500</td>
<td>435</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Steel</td>
<td>355</td>
<td>355</td>
<td>490</td>
<td>210</td>
<td>78.5</td>
<td>-</td>
</tr>
</tbody>
</table>

All steel to steel and steel to concrete connections are modelled as rigid. The boundary conditions at the outer supports are modelled to allow movement only in the longitudinal direction. Friction in the bearings is not accounted for. The two middle supports are constructed on piles where the soil stiffness is modelled using springs based on the settlements according to TRVR Bro 11. The static system of the footbridge is illustrated in figure 1.3.

The spring stiffness in the vertical direction is $1.955 \cdot 10^8 N/m$ and for the longitudinal direction $3.052 \cdot 10^7 N/m$. 

![Figure 1.3: Munktell - Static system, Elevation](image)
1.3.2 Bro 53-4-0105 Västberga

The second case is a pedestrian arch bridge designed by WSP according to the Swedish guidelines TRVK Bro 11 and TK Geo 11. The effective width is 5.0 m, the span is 40 m and the arch height is 6.3 m, see figure 1.4. A summary of the parts used in the model is presented in table 1.3 and the material parameters used in table 1.4.

![Figure 1.4: Västberga - Overview](image)

Table 1.3: Brief summary of parts in case Västberga.

<table>
<thead>
<tr>
<th>Part</th>
<th>Material</th>
<th>Element</th>
<th>Profile</th>
<th>Dimensions [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck</td>
<td>Steel S460</td>
<td>Shell</td>
<td>Plate</td>
<td>5.00·40.0  t = 0.01</td>
</tr>
<tr>
<td>Edge Beam</td>
<td>Steel S355</td>
<td>Beam</td>
<td>Rectangular, Hollow</td>
<td>0.25·0.40  t = 0.01</td>
</tr>
<tr>
<td>Arch</td>
<td>Steel S355</td>
<td>Beam</td>
<td>Rectangular, Hollow</td>
<td>0.25·0.45  t = 0.016</td>
</tr>
<tr>
<td>Bracing</td>
<td>Steel S355</td>
<td>Beam</td>
<td>Rectangular, Hollow</td>
<td>0.26·0.14  t = 0.0063</td>
</tr>
<tr>
<td>Lateral beam</td>
<td>Steel S355</td>
<td>Beam</td>
<td>Rectangular, Hollow</td>
<td>0.20·0.12  t = 0.016</td>
</tr>
<tr>
<td>Hanger</td>
<td>Steel S355</td>
<td>Beam</td>
<td>Rectangular, Hollow</td>
<td>0.25·0.25  t = 0.01</td>
</tr>
</tbody>
</table>

Table 1.4: Material properties for case Västberga.

<table>
<thead>
<tr>
<th>Material</th>
<th>$f_{yk}$ [MPa]</th>
<th>$f_{yd}$ [MPa]</th>
<th>$f_{uk}$ [MPa]</th>
<th>$E$ [GPa]</th>
<th>$\gamma$ [kN/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel S355</td>
<td>355</td>
<td>355</td>
<td>490</td>
<td>210</td>
<td>78.5</td>
</tr>
<tr>
<td>Steel S460</td>
<td>460</td>
<td>460</td>
<td>540</td>
<td>210</td>
<td>78.5</td>
</tr>
</tbody>
</table>

All connections are modelled as rigid. The boundary conditions at the supports are modelled to allow movement only in the longitudinal direction. Friction in the bearings is not accounted for. The static system of the footbridge is illustrated in figure 1.5.

![Figure 1.5: Västberga - Static system, Elevation](image)
Chapter 2

Theory

2.1 SDOF-Systems

Consider the simple spring system in figure 2.1

\[ f_s = ku \]  

(2.1)

where \( k \) is the spring stiffness, \( m \) is the mass, \( u \) is the mass displacement and \( p(t) \) is an external force. The system is assumed undamped and only allowed to move in one degree of freedom. The force in the spring is expressed as:

The inertia force from the mass can be expressed using Newtons 2nd law of motion:

\[ f_m = \dot{m} \dot{u} \]  

(2.2)

Equilibrium in figure 2.2 results in the well known equation of motion for an undamped single degree of freedom system:

\[ m\ddot{u} + ku = p(t) \]  

(2.3)
2.1.1 Free Vibration of an undamped system

Consider the system in figure 2.1. If the system is allowed to vibrate freely, i.e. \( p(t) = 0 \), the equation of motion becomes:

\[
m\ddot{u} + ku = 0 \tag{2.4}
\]

**Natural frequency**

Eq. 2.4 can be solved by assuming a sinusoidal solution [1]:

\[
u = \phi \sin(\omega t) \tag{2.5}
\]

where \( \phi \) is the magnitude of the free vibration, \( \omega \) is the angular frequency and \( t \) is the time. The second time derivative of the sinusoidal solution is:

\[
\ddot{u} = -\omega^2 \phi \sin(\omega t) = -\omega^2 u \tag{2.6}
\]

Then Eq. 2.4 becomes:

\[
-m\omega^2 u + ku = 0 \tag{2.7}
\]

\[
(k - \omega^2 m)\phi \sin(\omega t) = 0 \tag{2.8}
\]

\[
(k - \omega^2 m)\phi = 0 \tag{2.9}
\]

Considering only non-trivial solutions, i.e. \( \phi \neq 0 \), the problem is reduced to an eigenvalue problem. The angular frequencies \( \omega \) that satisfy Eq. 2.9 are called eigenfrequencies or natural frequencies \( \omega_n \) and the corresponding values \( \phi \) is the eigenmode of the system. The eigenfrequency can be expressed as:

\[
\omega_n = \sqrt{\frac{k}{m}} \tag{2.10}
\]

The physical interpretation of an eigenfrequency, or natural frequency, is the frequency at which the system vibrates during free vibration.

**General solution**

It is clear that both \( \cos(\omega_n t) \) and \( \sin(\omega_n t) \) satisfies the equation of motion, therefore the general solution can be expressed as [1]:

\[
u_h(t) = A\cos(\omega_n t) + B\sin(\omega_n t) \tag{2.11}
\]

where \( A \) and \( B \) are parameters that can be determined from the initial conditions. The following initial conditions are applied:

\[
u(0) = u_0 \quad \dot{u}(0) = v_0 \tag{2.12}
\]

This results in the following homogeneous solution to the differential equation:

\[
u_h(t) = u_0 \cos(\omega_n t) + \frac{v_0}{\omega_n} \sin(\omega_n t) \tag{2.13}
\]
2.1.2 Forced vibration of an undamped system

Recalling figure 2.1 and Eq. 2.3 but considering a harmonic load with the forcing frequency \( \omega \):

\[
m \ddot{u} + ku = p_0 \sin(\omega t)
\]  

(2.14)

The solution to this differential equation consists of the homogeneous solution derived for the case of free vibrations and a particular solution due to the harmonic load. A particular solution to this problem has the following form [1]:

\[
\dot{u}_p(t) = C \sin(\omega t)
\]  

(2.15)

\[
\ddot{u}_p(t) = -\omega^2 C \sin(\omega t)
\]  

(2.16)

Inserting Eq. 2.15 and Eq. 2.16 in Eq. 2.14 and using \( \omega_n^2 = k/m \):

\[
C = \frac{p_0}{k} \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}
\]  

(2.17)

The particular solution becomes:

\[
\dot{u}_p(t) = \frac{p_0}{k} \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \sin(\omega t) \quad \omega \neq \omega_n
\]  

(2.18)

From Eq. 2.18 it is evident that the particular solution for an undamped system approaches infinity when the forcing frequency approaches the natural frequency.

The general solution to the case of forced vibration:

\[
u(t) = u_h(t) + u_p(t)
\]  

(2.19)

\[
u(t) = u_0 \cos(\omega_n t) + \frac{v_0}{\omega_n} \sin(\omega_n t) + \frac{p_0}{k} \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \sin(\omega_t)
\]  

(2.20)

From Eq. 2.19 and Eq. 2.20 it is clear that the homogeneous solution does not depend on the forcing frequency, only on the natural frequency \( \omega_n \). This is called the transient solution. The particular solution depends on the forcing frequency \( \omega \) and increases when the forcing frequency approaches the natural frequency. This is called the steady-state response of the system. The solution is illustrated in figure 2.3.

![Figure 2.3: Steady-state response and total response for an undamped system, \( \omega_n = 2\omega \).](image-url)
2.1.3 Free Vibration of a damped system

Consider the damped spring system in figure 2.4

where $k$ is the spring stiffness, $c$ is the viscous damping of the system, $m$ is the mass of the system, $u$ is the displacement of the mass and $p(t)$ is an external force.

The viscous damping model accounts for energy loss through friction and thermodynamic effects by introducing a damping parameter $c$ and is based on a linear relation between the damping and the time derivative of the displacement $\dot{u}$. This can be added to the previously derived equation of motion for undamped systems.

$$m\ddot{u} + c\dot{u} + ku = p(t) \quad (2.21)$$

Considering free vibration once again, i.e $p(t) = 0$ the equation is expressed as:

$$m\ddot{u} + c\dot{u} + ku = 0 \quad (2.22)$$

By rearranging the expression, introducing the term $\xi = c/2m\omega_n$ called the critical damping ratio and recalling $\omega_n^2 = k/m$ the equation can be rewritten as:

$$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = 0 \quad (2.23)$$

General solution

The homogenous solution to this equation is solved by introducing an exponential solution and making use of initial conditions [1]. Complete derivation will not be presented here, only the solution, the reader is referred to literature on the subject for further reading.

$$u_h(t) = e^{-\xi \omega_n t} \left[ u_0 \cos(\omega_D t) + \left( \frac{v_0 + \xi \omega_n u_0}{\omega_D} \right) \sin(\omega_D t) \right] \quad (2.24)$$

where

$$\omega_D = \omega_n \sqrt{1 - \xi^2} \quad (2.25)$$

$\omega_D$ is the natural frequency for damped vibrations and is for small values on the critical damping ratio $\xi$ close to the natural frequency for undamped vibrations. Therefore $\omega_D \approx \omega_n$ is assumed for practical applications.
### 2.1.4 Forced vibration of a damped system

For the case of harmonic loading Eq. 2.23 is written as:

\[ \ddot{u} + 2\xi \omega_n \dot{u} + \omega_n^2 u = p_0 \sin(\omega t) \]  

(2.26)

**General solution**

As for the case of an undamped system the solution consists of a homogeneous solution and a particular solution. The homogeneous solution was derived for the case of free vibration, see Eq. 2.24. As for the homogenous part of the solution the reader is referred to litterature on the subject for complete derivation [1].

\[ u_p(t) = C \sin(\omega t) + D \cos(\omega t) \]  

(2.27)

where

\[ C = \frac{p_0}{k} \left[ \frac{1 - (\omega/\omega_n)^2}{1 - (\omega/\omega_n)^2 + [2\xi(\omega/\omega_n)]^2} \right] \]  

(2.28)

\[ D = \frac{p_0}{k} \left[ \frac{-2\xi \omega / \omega_n}{1 - (\omega/\omega_n)^2 + [2\xi(\omega/\omega_n)]^2} \right] \]  

(2.29)

The complete solution consists of Eq. 2.24 and Eq. 2.27 and is expressed as:

\[ u(t) = u_h(t) + u_p(t) \]  

(2.30)

From Eq. 2.27 - 2.29 it is clear that the particular solution has the same angular frequency as the forcing frequency \( \omega \), the parameters C and D govern the magnitude of the solution and depend on the damping ratio and the proximity of the forcing frequency to the natural frequency. This is called the steady-state response. From Eq. 2.24 it is clear that the homogeneous solution has the same angular frequency as the natural frequency for damped vibrations \( \omega_n \). This is called the transient solution. Due to the first term of the transient solution \( e^{-\xi \omega_n t} \), which decreases with time \( t \), the total response will converge towards the steady-state response with time. The general solution for the forced vibration with damping is presented in figure 2.5.

![Figure 2.5: Steady-state and total response for a damped system, \( \omega_n = 2\omega \).](image)

**2.1.5 Dynamic response factor**

In order to illustrate the effects from dynamic loading during resonance, \( \omega = \omega_n \), the dynamic response factor is introduced as the relation between the deformation during static loading \( u_0 = p_0/k \) and the deformation during steady-state \( u_{st} \).

\[ R_d = \frac{u_0}{u_{st}} \]  

(2.31)

where \( u_{st} \) is the amplitude of the particular solution.
2.1. SDOF-SYSTEMS

CHAPTER 2. THEORY

This relation serves as an amplification factor of the static deformation due to resonance.

Since any sum of sine and cosine can be expressed as:

\[A \sin(\theta) \pm B \cos(\theta) \equiv R \sin(\theta + \alpha)\]  \hspace{1cm} (2.32)

where the resulting amplitude \(R = \sqrt{A^2 + B^2}\) and the phase shift \(\alpha = \arctan(B/A)\). The particular solution in Eq. 2.27 - 2.29, becomes:

\[u(t) = u_0 \sin(\omega t - \theta) = R \frac{p_0}{k} \sin(\omega t - \theta)\]  \hspace{1cm} (2.33)

where \(u_0 = \sqrt{C^2 + D^2}\) and \(\theta = \arctan(D/C)\). With this expression for the amplitude of the steady state solution and recalling the static deformation to be \(p_0/k\) an expression for the dynamic response factor can be acquired [1].

\[R_d = \frac{1}{\sqrt{1 - \left(\frac{\omega}{\omega_n}\right)^2 + \left[2\xi \left(\frac{\omega}{\omega_n}\right)\right]^2}}\]  \hspace{1cm} (2.34)

\[\theta = \tan^{-1} \frac{2\xi \left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\]  \hspace{1cm} (2.35)

The dynamic response factor \(R_d\) is visualized in figure 2.6 for a variety of different values on the critical damping ratio \(\xi\).

Figure 2.6:
Dynamic response factor plotted as a function of forcing frequency.

An expression for the maximum response in terms of acceleration can be derived by differentiating Eq. 2.33.

\[\ddot{u}(t) = R_d \frac{p_0}{k} \sin(\omega t - \theta)\]  \hspace{1cm} (2.36)

\[\ddot{u}(t) = \frac{\omega^2}{\omega_n^2} R_d \frac{p_0}{\sqrt{km}} \cos(\omega t - \theta)\]  \hspace{1cm} (2.37)

\[\ddot{u}(t) = -\left(\frac{\omega}{\omega_n}\right)^2 R_d \frac{p_0}{m} \sin(\omega t - \theta)\]  \hspace{1cm} (2.38)

The maximum response from Eq. 2.38 is acquired when the sine function equals 1. Further, the relation \(f/f_n = \omega/\omega_n\) and Eq. 2.34 is used. The maximum accelerations are:

\[
\ddot{u}_{max} = -\left(\frac{f}{f_n}\right)^2 \frac{p_0}{m} \frac{1}{\sqrt{1 - (f/f_n)^2}^2 + [2\xi (f/f_n)]^2}}\]  \hspace{1cm} (2.39)
2.2 Generalized SDOF-Systems

So far only idealized systems with a single degree of freedom have been considered, however, it is possible to reduce more complex systems to SDOF - systems as well, these systems are called generalized SDOF - systems. This is convenient since the equations derived for SDOF - systems holds for general SDOF - systems as well. This is achieved by assuming a shape function \( \phi \) for the system and applying the principle of virtual work to the equation of motion, complete derivations can be found in most literature on the subject. The following expression is acquired [1].

\[
\hat{m} \ddot{u} + \hat{c} \dot{u} + \hat{k} u = \hat{p}(t) \tag{2.41}
\]

where the generalized quantities are

\[
\hat{m} = \int_0^L m(x) \phi^2(x) \, dx \tag{2.42}
\]
\[
\hat{c} = \int_0^L c(x) \phi'(x)^2 \, dx \tag{2.43}
\]
\[
\hat{k} = \int_0^L EI(x) \phi''(x)^2 \, dx \tag{2.44}
\]
\[
\hat{p}(t) = \int_0^L p(x,t) \phi(x) \, dx \tag{2.45}
\]

For the case of a sinusoidal response \( \phi(x) = \sin(x \pi / L) \) Eq. 2.42 - 2.45 can be simplified. It is convenient to express the generalized mass and force as a simplified relation applicable by hand calculation.

**Generalized Mass**

Assuming an evenly distributed mass \( m(x) \) on the length \( L \) expressed as a constant mass per unit length \( \bar{m} \) where \( \bar{m} \cdot L = m \), Eq 2.42 is simplified:

\[
\hat{m} = \bar{m} \int_0^L \sin^2 \left( \frac{x \pi}{L} \right) \, dx = \bar{m} \left[ \frac{x}{2} + \frac{\sin \left( \frac{x \pi}{L} \right)}{4} \right]_0^L = \bar{m} \cdot L = \frac{m}{2} \tag{2.46}
\]

It seems from Eq. 2.46 that for a simply supported beam and a sinusoidal response the generalized mass is half of the total mass. This relation can be shown to hold for higher sinusoidal modes as well.

\[
\hat{m} = \frac{m}{2} \tag{2.47}
\]

**Generalized Force**

Assuming an evenly distributed and time dependant load \( q(t) \) on the length \( L \) expressed as a constant distributed load per unit length \( \bar{q}(t) \) where \( \bar{q}(t) \cdot L = p(t) \),

\[
\hat{p}(t) = \bar{q}(t) \cdot \int_0^L \sin \left( \frac{x \pi}{L} \right) \, dx = -\bar{q}(t) \left[ \frac{L}{\pi} \cos \left( \frac{x \pi}{L} \right) \right]_0^L = 2 \cdot \frac{\bar{q}(t) \cdot L}{\pi} = \frac{2}{\pi} \cdot p(t) \tag{2.48}
\]

where \( p(t) = q(t) \cdot A \).

It seems from Eq. 2.47 that for a simply supported beam subjected to an evenly distributed load \( q(t) \) the generalized force \( \hat{p}(t) \) can be expressed as:

\[
\hat{p}(t) = \frac{2}{\pi} \cdot p(t) \tag{2.49}
\]
2.2.1 Modal Expansion

The numerical approach commonly implemented in a FE-software is to consider generalized SDOF-systems using modal expansion. Given the equation of motion for a system of multiple degrees of freedom (MDOF) [1]:

\[ m \ddot{u} + c \dot{u} + k u = p(t) \]  \hspace{1cm} (2.50)

where \( m \) is the mass matrix, \( k \) is the stiffness matrix, \( c \) is the damping matrix, \( u \) is the displacement vector and \( p(t) \) is a force vector.

The basis of modal expansion is that a vector of order \( N \) can be represented using \( N \) independent vectors [1]. Expressing the dynamic behaviour of an arbitrary system only in terms of independent vectors allows for simplified calculations and faster computing. By representing the response \( u \) of an object in terms of a linear combination of its natural modes \( \phi \) a fair approximation can be acquired from only a few modes, this is called modal truncation. This is commonly expressed as [1]

\[ u = \sum_{r=1}^{N} \phi_r q_r = \phi q \]  \hspace{1cm} (2.51)

where \( \phi_r \) is the natural mode, or eigenvector, and \( q_r \) is a multiplier governing the magnitude of the current mode in the linear combination. For complete derivation and proofs the reader is again referred to literature on the subject.

Applying Eq. 2.51 to Eq. 2.50:

\[ M \ddot{q} + C \dot{q} + K q = p(t) \]  \hspace{1cm} (2.52)

Eq. 2.52 is multiplied with the transpose of the eigenvector \( \phi^T \):

\[ \phi^T M \ddot{q} + \phi^T C \dot{q} + \phi^T K q = \phi^T p(t) \]  \hspace{1cm} (2.53)

\[ M \phi \ddot{q} + C \phi \dot{q} + K \phi q = p(t) \]  \hspace{1cm} (2.54)

Eq. 2.54 contains a set of \( N \) independent equations. This is the equation of motion on uncoupled form containing the generalized mass, damping, stiffness and force. The uncoupling requires the use of classical damping such as Rayleigh damping, however, non-classical damping is not covered in this report, therefore the reader is referred to literature on the subject. The total response is a linear combination of the response for each mode included in the analysis.

For a certain mode \( m \) Eq. 5.54 becomes

\[ M_m \ddot{q}_m + C_m \dot{q}_m + K_m q_m = P_m(t) \]  \hspace{1cm} (2.55)

See Eq. 2.41 - 2.45 for comparison. The major difference compared to the more analytical generalized SDOF-system is the use of natural modes instead of assumed modeshapes.

For a given mode \( m \) the relation between the displacement vector \( u_m \) and the generalized displacement \( q_m \) in Eq. 2.55 can based on Eq. 2.51 be written as

\[ u_m = \phi_m q_m \]  \hspace{1cm} (2.56)

Based on Eq. 2.56 the simplified equation used for determining the maximum peak acceleration response in Eq. 2.40 is expressed using generalized quantities

\[ u_{max, m} = \phi_m \left( - \frac{P_m}{M_m} \frac{1}{2 \xi_m} \right) \]  \hspace{1cm} (2.57)

where \( u_{max, m} \) is a vector containing the maximum peak acceleration for all nodes given the mode \( m \). Expressed for a node \( n \) Eq. 2.57 becomes

\[ u_{max,m,n} = - \mu_{m,n} \frac{P_m}{M_m} \frac{1}{2 \xi_m} \]  \hspace{1cm} (2.58)

where \( \mu_{m,n} \) is the modal value for the node \( n \) in the eigenvector \( \phi_n \).
2.3 Damping models

The damping of a structure is the phenomenon where the kinetic energy during vibration dissipates and disperse. This results in a reduction of kinetic energy over time. There are several mechanisms governing the damping of a structure such as dissipation in the connections and joints of the structure and dispersion within the material due to internal friction and non-linear behaviour. Because of the highly complex nature of each of these mechanisms it is convenient to adopt some idealized model based on measurements of similar structures.

Recalling Eq. 2.23 from the case of free vibrations of a damped system:

\[ \ddot{u} + 2\xi \omega_n \dot{u} + \omega_n^2 u = 0 \]  \hspace{1cm} (2.59)

where the term \( \xi = c/c_{cr} \) is commonly referred to as the critical damping ratio and \( c_{cr} = 2\sqrt{mk} \). If \( c < c_{cr} \) the system will oscillate, if \( c > c_{cr} \) the system will not oscillate but return directly to its state of equilibrium and if \( c = c_{cr} \) the system returns to equilibrium in the most direct manner. From figure 2.7 it is clear that \( \xi < 1.0 \) results in an oscillating system while \( \xi > 1.0 \) is prevented from oscillating. Furthermore \( \xi = 1.0 \) returns to equilibrium more quickly than \( \xi = 2.0 \).

![Figure 2.7: Transient solution of a damped system for a variety of values of the critical damping ratio \( \xi \).](image)

2.3.1 Modal Damping

The concept of critical damping ratio as introduced in previous sections is based on damping independent of frequency. Through vibration tests it has been shown that the damping generally varies with the forcing frequency in the case of forced vibrations [1]. Therefore it is necessary to define damping ratios with respect to frequency. In the case of existing structures damping ratios can be acquired from dynamic testing where each frequency corresponds to a damping ratio. These measurements could in theory be applied to future structures of similar properties, but for practical use it is convenient to define an idealized damping model that accounts for the variation in frequency.

Analogues to the SDOF - system the critical damping ratio is defined as

\[ \xi_m = \frac{C_m}{2M_m \omega_m} \]  \hspace{1cm} (2.60)

where \( \omega_m = \sqrt{K_m/M_m} \) is the natural frequency for the mode \( m \).
2.3. DAMPING MODELS

2.3.2 Rayleigh Damping

A common method to define the generalized viscous damping parameter \( C_m \) is by assuming a mass-proportional and stiffness-proportional damping \([1]\). The most common model is the Rayleigh damping defined as:

\[
C_m = a_0 M_m + a_1 K_m
\]  \hspace{1cm} (2.61)

Recalling the definition of the critical damping ratio from Eq. 2.60.

\[
C_m = 2 \xi_m M_m \omega_m
\]  \hspace{1cm} (2.62)

Inserting Eq. 2.62 in Eq. 2.61 and using \( \omega_m^2 = K_m / M_m \) the following expression is acquired

\[
\xi_m = \frac{a_0}{2 \omega_m} + \frac{a_1 \omega_m}{2}
\]  \hspace{1cm} (2.63)

The variation of damping with regard to natural frequency is illustrated in figure 2.8.

![Figure 2.8: Rayleigh damping.](image)

Writing Eq. 2.63 in matrix form for the natural modes i and j gives:

\[
\begin{bmatrix}
\xi_i \\
\xi_j
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
1/\omega_i & a_0 \\
1/\omega_j & a_1
\end{bmatrix} \begin{bmatrix}
a_0 \\
a_1
\end{bmatrix}
\]

In this form it is clear that the Rayleigh parameters \( a_0 \) and \( a_1 \) can be determined experimentally by measuring the damping ratio \( \xi_i \) and \( \xi_j \) for each natural mode of interest \( \omega_i \) and \( \omega_j \). The damping ratio can be quantified from measurements with the "half-bandwidth" method which can be found in most literature on the subject of dynamics.

2.3.3 Design Implementation

Even though the use of modal damping may capture the behaviour of a structure fairly accurately the parameters are unique to the structure considered. In some situations these parameters can be based on similar structures, but in most cases the uncertainties are too significant for it to be useful in the design phase of a bridge. However, during a dynamic analysis of an existing structure the Rayleigh damping is a useful tool to capture and model the damping accurately.

For most practical cases the design values used are in terms of critical damping ratio \( \xi \) where these values usually are based on previous structures and provided in a table as figure 2.9.
CHAPTER 2. THEORY

2.4 Response

A central part when considering the dynamic comfort requirements is to quantify and evaluate the response generated by pedestrian activity. The aim is to evaluate the response in a way that coincides with the perceived comfort level. The by far most supported model of evaluation is in terms of acceleration, it seems to coincide well with the perceived comfort experienced by a pedestrian. However, it is worth noting that the perceived comfort differs significantly between pedestrians which further increases the complexity of the problem.

2.4.1 Quantification

A measured footfall is characterized by a large degree of randomness. The non-deterministic behaviour of the time history presented in figure 2.10 is not possible to capture as a simple function since both the amplitude and frequency varies in a seemingly random manner. For analytical purpose a response of this type could be evaluated through a Fast Fourier Transform (FFT) in order to determine the natural frequencies of the system. But for this case, where the aim is to compare measured vibrations to human comfort limits, it is usually more convenient to assign a single value to any measured response.

\[ x_{RMS} = \sqrt{\frac{1}{T} \int_{t}^{t+T} x^2 \, dx} \]  

where \( T \) is the time period of the measured response. It is clear that the choice of time period has a significant impact on the RMS, it is therefore important to choose a representative sample of the measured response. There are guidelines and recommendations on the subject of RMS, however this mostly covers comfort requirements in buildings and is therefore not covered by this report.
2.4. RESPONSE

As a design method where a harmonic load model is applied the response shows a much more deterministic behaviour, see figure 2.11. Therefore the peak value is conveniently used in these situations. Assuming a sinusoidal response it is convenient to determine the relation of RMS and Peak value. Applying Eq. 2.64 to a sine function \( \sin(x) \) for the period \( 0 - 2\pi \)

\[
a_{\text{rms}} = \sqrt{\frac{1}{T} \int_{t}^{t+T} \sin^2(x) dx} = \sqrt{\frac{1}{2\pi} \left[ \frac{x}{2} + \frac{\sin(x)}{4} \right]_0^{2\pi}} = \sqrt{\frac{\pi}{2\pi}} = \frac{1}{\sqrt{2}} \quad (2.65)
\]

The relation in Eq. 2.65 is further illustrated in figure 2.12.

![Figure 2.12: Comparison of Peak value and RMS for \( \sin(\theta) \).](image)

2.4.2 Evaluation

By quantifying accelerations a certain response is given a certain value. To evaluate the response it is compared to some reference values based on human perception. Experiments have shown a dependency between experienced comfort due to accelerations and the frequency at which these accelerations occur. A design limit presented by J. Blanchard based on measurements by Leonard and Smith defines the acceptable accelerations in terms of RMS as a function of frequency \( [7] \).

\[
a_{\text{limit}} = 0.5 \sqrt{f} \quad (2.66)
\]

This limit is currently used by the British standard BS 5400 as a design limit.

Another important factor for determining the comfort requirements is the activity of the subject and the direction of the vibrations. Higher accelerations are generally tolerated during walking compared to standing or sitting down. Therefore different limits are set depending on the activity, frequency and direction \([8]\). The reference curves in figure 2.13 and 2.14 can be found in the document SS-ISO 10137:2008 and are used for the serviceability of structures. It represents the limit of perceived RMS accelerations by a human.
In order to implement these reference curves in a design situation where the accelerations are represented as peak values, the curves are adjusted with the relation derived in Eq. 2.65 and is presented in figure 2.15 and 2.16.

Several other limits have been proposed based on experiments by a variety of authors [7]. A summary presented as a literature review in the Journal of Sound and Vibration by S. Zicanovic, A. Pavic, P. Reynolds is shown in figure 2.17.
The large variation of proposed acceleration limits depends on the difficulty in quantifying something as subjective as comfort limits. The perceived comfort clearly differs between humans. Furthermore, the perception of acceptable accelerations may differ due to the visual input and expected behaviour of a structure. In an experiment presented in a report by Hivoss two pedestrian bridges with similar properties in terms of stiffness, mass and response but different in appearance were evaluated by a large number of pedestrians [4]. It turned out that a much larger portion of the pedestrians in the experiment found the visually sturdier bridge disturbing, while the visually lighter bridge was described in a much larger extent as amusing and exciting. This is explained by the expectation a pedestrian has simply through visual input. A more slender looking bridge is expected to have some dynamic response, it is therefore not as disturbing to the pedestrian as a sturdier looking bridge. See figure 2.18.
Chapter 3

Pedestrian Loads

In order to simulate the response from a pedestrian walking or running it is necessary to represent the footfall load in an accurate yet simple manner. To establish a general model for the pedestrian footfall has been the aim of many authors throughout the years with a great variation in proposed models and parameters [7]. This is largely due to the number of different methods to measure and model a general footfall but also due to the complexity of pedestrian behaviour. The statistical diversity among pedestrians gives rise to two different approaches. The probabilistic load model where the statistical distribution is considered for each pedestrian and the deterministic load model where a general model is established based on an average pedestrian and assumed to be applicable to groups of pedestrians.

It was discovered only recently that the behaviour of pedestrians is even more complex than previously thought due to interaction effects [3][7]. During the opening of the Millennium bridge in London large lateral vibrations were observed, it was explained through lateral synchronization effects which results in a much higher probability that a pedestrian will walk in such a way that the lateral mode is excited, this has also been referred to as "lock-in" effects.

3.1 The deterministic load model

The most common way to represent dynamic loads is the deterministic force model where the footfall is represented using a Fourier series of $H$ number of harmonics $h$:

$$F_p(t) = G + \sum_{h=1}^{H} G\alpha_h \sin(2\pi f_w t - \phi_h)$$  \hspace{1cm} (3.1)

The weight of a pedestrian $G$ is set to 700 N, the coefficients $\alpha_h$ are parameters determined through measurements (also referred to as dynamic load factor, or DLF), the forcing frequency $f_w$ is the walking frequency of the pedestrian, $h$ is the harmonic number and $\phi_h$ is the phase shift of the harmonic. Note that the first term in the series is the static self weight of a pedestrian and is only present in vertical load models.

3.1.1 Harmonics

A harmonic is a positive whole number integer multiple of the fundamental frequency, where the fundamental frequency in this case is the walking frequency. If the walking frequency is $2Hz$ the first three harmonics would be $4Hz$, $6Hz$ and $8Hz$. The use of harmonics in the deterministic model is based on measurements of a pedestrian footfall. The general footfall consists of several harmonics added together to produce a time history diagram that captures the measured footfall in a representative manner, see figure 3.1.

From figure 3.1 it is clear that only using the first harmonic yields in a purely sinusoidal load and by increasing the number of harmonics a more complex behaviour can be represented.
3.1. THE DETERMINISTIC LOAD MODEL

CHAPTER 3. PEDESTRIAN LOADS

3.1.2 Dynamic load factor

The dynamic load factor is the relation of the static and dynamic weight of the pedestrian for a certain harmonic, load direction and activity. Dynamic load factors have been measured at many occasions but the by far most extensive measurements on the subject of vertical loads was made by Stuart Kerr in 1998 where footfall loads were measured 882 times by several different subjects during walking in the frequency range of 1.0 - 2.8 Hz, see figure 3.2 a-d.

Although these measurements are extensive, the test itself was performed on a stationary surface [7]. Later measurements have shown lower dynamic load factors when measured on a full scale bridge. This is explained by the lower stiffness of a bridge compared to a floor. The measurements on a rigid surface may overestimate the response when applied to a footbridge, however, it serves as an upper limit to the dynamic load factor since generally the stiffness is be assumed to be lower than a rigid floor.
CHAPTER 3. PEDESTRIAN LOADS

3.1. THE DETERMINISTIC LOAD MODEL

3.1.3 Vertical Loads

Some proposed dynamic load models for walking are presented in table 3.1.

Table 3.1: Vertical dynamic load models [7]

<table>
<thead>
<tr>
<th>DLF</th>
<th>Young &amp; Kerr</th>
<th>Schulze</th>
<th>Bachmann [2 Hz]</th>
<th>Bachmann [2.4 Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.37($f - 0.95$) $\leq$ 0.56</td>
<td>0.37</td>
<td>0.40</td>
<td>0.50</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$0.054 + 0.0044f$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>$0.026 + 0.005f$</td>
<td>0.12</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>$0.010 + 0.0051f$</td>
<td>0.04</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>-</td>
<td>0.08</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The footfall of a single pedestrian can be illustrated by plotting the Fourier series for certain parameters. The examples shown in figure 3.3 and 3.4 are footfall models proposed by Schulze and Bachmann plotted for the walking frequency 2 Hz.

![Figure 3.3: Vertical load model proposed by Schulze at 2 Hz](image)

![Figure 3.4: Vertical load model proposed by Bachmann at 2 Hz](image)

3.1.4 Lateral Loads

The lateral loads due to walking are induced by a pedestrian shifting his or her body mass when taking a step, see figure 3.5.

![Figure 3.5: Description of lateral forces during walking](image)
3.1. THE DETERMINISTIC LOAD MODEL

Unlike vertical loads, lateral loads are acting in half of the walking frequency since the force component is acting in a right to left manner. Another difference is the absence of static weight from the pedestrian. Load models have been proposed in a similar manner as vertical load models but not to the same extent. Two proposed models are presented in table 3.2.

Table 3.2: Lateral dynamic load models [7].

<table>
<thead>
<tr>
<th></th>
<th>DLF</th>
<th>Schulze</th>
<th>Bachmann</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.039</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.010</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.043</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.012</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>0.015</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The footfall of a single pedestrian is illustrated as for vertical loads by plotting the Fourier series, see figure 3.6 and 3.7.

Figure 3.6: Lateral load model proposed by Schulze at 2 Hz
Figure 3.7: Lateral load model proposed by Bachmann at 2 Hz

3.1.5 Longitudinal Loads

The longitudinal component behaves similar to the vertical where the excitation is in the same frequency as the walking frequency. A few longitudinal dynamic load factors are presented in table 3.3.

Table 3.3: Longitudinal dynamic load models [7].

<table>
<thead>
<tr>
<th></th>
<th>DLF</th>
<th>Schulze</th>
<th>Bachmann</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.037</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.204</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.026</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.083</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>0.024</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The footfall of a single pedestrian is illustrated for longitudinal loads by plotting the Fourier series, see figure 3.8 and 3.9.
3.1.6 Measured footfalls

The pedestrian footfall has been measured at many different occasions. The average walking frequency is around 2 Hz with a standard deviation of $0.175 - 0.220$ Hz depending on the measurements [7]. The most common method of capturing the pedestrian footfall involves pressure plates that subjects walk upon. This results in captured time history diagrams such as the diagrams presented in figures 3.10 and 3.11.

A few more diagrams are presented in figure 3.12 where the footfall is illustrated for a variety of activities.
3.1. THE DETERMINISTIC LOAD MODEL

CHAPTER 3. PEDESTRIAN LOADS

Figure 3.12:
Time history diagram of footfalls for a variety of activities [3].

An average footfall during walking is presented in figure 3.13.

Figure 3.13:
Average footfall during walking [3].

From figure 3.13 and 3.12 it is clear that a single human footfall during walking is characterized by a saddle shape. The first peak in the footfall corresponds to the impact of the heel while the other maximum corresponds to the thrust of the foot. It is this behaviour that is attempted to capture using the harmonics and dynamic load factors in the Fourier series, see section 3.1, 3.1.1 and 3.1.2.
3.2 Pedestrian characteristics

It is now necessary to consider a situation with several pedestrians crossing a footbridge. Each pedestrian will have certain characteristics such as mass, speed and frequency and since each pedestrian enters the bridge at a different moment there will be an initial phase shift in the walking frequency as well. The situation proves even more difficult due to human behaviour, it has been shown that pedestrians tend to modify their walking depending on the situation. From this large variation in characteristics it is of interest to estimate, based on reasonable assumptions, the maximum number of pedestrians in synchronization with each other and the footbridge in order to establish a simplified load model.

The approach described by Sétra involves comprehensive simulations of randomly generated crowds consisting of N number of pedestrians using Monte Carlo algorithms. These pedestrians are simulated to walk with the speed of 1.5 m/s across a footbridge with the critical damping ratio $\xi$ [3]. Each pedestrian is given a completely random phase shift and a random walking frequency based on normal distribution. The walking frequency is centred around the natural frequency of the footbridge with a standard deviation of 0.175 Hz. The response generated by this crowd of N pedestrians is computed in terms of peak accelerations and the maximum value over a sufficiently long period of time is noted as $a_{\text{max}}$, see figure 3.14. The number of evenly distributed pedestrians perfectly synchronized with the footbridge required to generate the same maximum response $a_{\text{max}}$ is calculated and defined as the equivalent number of pedestrians $N_{\text{eq}}$, see figure 3.15.

![Figure 3.14](image1.png)

![Figure 3.15](image2.png)

This simulation is performed for a fixed value on the critical damping ratio $\xi$ and the number of pedestrians $N$ and repeated 500 times. The number of equivalent pedestrians is determined as the 95% characteristic value, see figure 3.16.
By repeating this process for varying number of pedestrians and critical damping ratio a relation describing the equivalent number of pedestrians is acquired.

\[ N_{eq} = 10.8 \sqrt{\xi N} \]  

(3.2)

Or if expressed as a crowd density \( d \) using \( d = N/S \) where \( S \) is the surface area of the bridge deck.

\[ d_{eq} = d \cdot 10.8 \sqrt{\frac{\xi}{N}} \]  

(3.3)

The assumption of normally distributed walking frequency and random phase shift relies on all pedestrians behaving independently of one another. It was shown during the dynamic analysis of the Millennium bridge in London after the opening that a large number of pedestrians were walking with the same frequency and phase compared to what would be expected from random number analysis. This was explained by the high pedestrian density, at a certain point a single pedestrian is no longer able to move freely but has to adapt his or her walking to other pedestrians. This yielded in a larger amount of synchronized pedestrians than a statistical approach of random walking frequency and random phase would generate. These effects are called vertical synchronization.

### 3.2.1 Vertical Synchronization effects

The method to account for these effects used by Sétra is to assume complete synchronization of walking frequency but phase shift still completely random as a design situation [3]. Based on this assumption the simulations explained in the previous subsection can be repeated and an expression for the equivalent number of pedestrians \( N_{eq} \) is acquired. These effects have been shown to occur at pedestrian densities over 1 pedestrian/m² [3]. See figure 3.17 for various crowd densities.

For a very dense crowd where vertical synchronization effects are present:

\[ N_{eq} = 1.85 \sqrt{N} \]  

(3.4)

Or if expressed as a crowd density \( d \) using \( d = N/S \):

\[ d_{eq} = d \cdot 1.85 \sqrt{1/N} \]  

(3.5)
The equivalent pedestrian density with respect to pedestrian density and critical damping ratio can be visualized by plotting Eq. 3.3 and Eq. 3.5 in the range of 0.0 - 1.5 pedestrians/m², see figure 3.18. There is a clear increase in equivalent pedestrian density at densities above 1 pedestrian/m² due to vertical interaction effects. Furthermore, it seems that a higher value on the critical damping ratio results in higher synchronization effects in the range 0.0 - 1.0 pedestrian/m².

**Figure 3.18:**
Equivalent pedestrian density as a function of pedestrian density and critical damping ratio $\xi$.

### 3.2.2 Lateral synchronization effects

Another effect recently observed is that during large lateral vibrations pedestrians would seem to favour a walking frequency that coincides with the optimal excitation frequency of the current lateral mode. It seems that at a certain horizontal acceleration a pedestrian starts compensating for the vibrations by shifting the body mass to counteract the vibrations which in turn further excites the lateral vibrations. These synchronization effects were observed at the opening of the Millennium bridge in 2000 and was later determined to be the cause of the large lateral vibrations. This is called lateral synchronization or "Lock-in". It was proposed by Dallard et. al. and adopted by Arup that lock in effects occur at a critical number of pedestrians described by the relation [7]

$$N_L = \frac{8\pi c f_n M}{k}$$

where $f_n$ is the natural frequency for the current lateral eigenmode, M the corresponding modal mass, $c$ the modal damping ratio and $k$ a parameter describing the walking force. During the dynamic analysis of the Millennium bridge by Arup it was found that the parameter $k = 300Ns/m$ within the frequency range 0.5 - 1.0 Hz.

The approach used by Séttra assumes that lock in effects occur at a certain magnitude on the lateral vibrations [3]. Tests have been performed on the pedestrian bridge Solférino where the lateral accelerations and synchronization rates were recorded for different streams of pedestrians. An example of the performed tests is shown in figure 3.19. During this test the bridge was subjected to an increasing crowd load over time.
3.3. OTHER PROPOSED LOAD MODELS

The rapid increase in lateral acceleration and synchronization rate at $t > 500s$ indicates the limit at which lateral synchronization effects begin to appear. The corresponding lateral accelerations are $0.15 - 0.20 \text{ m/s}^2$. Similar tests all resulted in lateral synchronization at $0.10 - 0.20 \text{ m/s}^2$. As a design guideline it is suggested that lateral accelerations should be limited to $0.10 \text{ m/s}^2$ in order to avoid these synchronization effects [3].

3.3 Other proposed load models

3.3.1 Probabilistic load model

The load models used in this report are the deterministic models where a footfall is defined as a Fourier series and pedestrians are assumed to always produce identical loads. Interaction effects are accounted for by considering an equivalent number of pedestrians that are completely synchronized with each other and evenly distributed across the bridge.

A different approach is the probabilistic load model where the variation of each footfall is accounted through a probability distribution. This method relies heavily on a large amount of measurements in order to predict the variation of footfalls produced by pedestrians in different types of situations. In a probabilistic load model interaction effects would not be considered by introducing an equivalent number of pedestrian but would be included in the load model for a single pedestrian. A reliable estimates of crowd loading would be acquired by simply combining force models from single pedestrians. However, this load model is difficult to apply in practice due to the scarcity of measurements for pedestrian induced loads on footbridges, it is therefore not covered further in this report.
3.3.2 Walking force model

A model proposed by Blanchard et al. is the walking force model where a concentrated harmonic force is moving across the footbridge with a velocity $v = 0.9 f_n$ and the force $F(t)$, see figure 3.20 [7].

$$F(t) = 180\sin(2\pi f_n t)$$  \hspace{1cm} (3.7)

where $f_n$ is the natural frequency of the footbridge. The magnitude of the force corresponds to a dynamic load factor of 0.257.

This is considered to be one of the earliest attempts to define a load model applicable as a design tool and is the load model adopted by the British design guidelines BS 5400. However, the model only includes the first harmonic and thus cannot capture effects of higher harmonics. Furthermore, the dynamic load factor is significantly lower than other load models presented later on, see table 3.1.

The use of a moving load further reduces the applicability of this model since it requires complicated calculations to simulate the moving load. Other shortcomings of this model have been identified as well [7]. This load model will therefore not be included in this report any further.
Chapter 4

Methods

4.1 Current Guidelines

The theory and terminology for predicting the response due to pedestrian loads have been presented and explained in previous sections. Due to the recentness of dynamic effects on footbridges the guidelines on the subject offer only limited help. A few guidelines currently available are summarized here for the purpose of comparison with the methods and recommendations presented by Willford & Young and Sétra.

4.1.1 Bro 2004

In the document Bro 2004 by the Swedish road administration Vägverket, a method for designing footbridges with regard to footfall induced vibrations is proposed. A concentrated stationary harmonic force $F$ is placed at a point on the deck of the bridge where the largest vertical accelerations are generated [12]. The force $F$ is defined as:

$$F = k_1 k_2 \sin(2\pi f_F t)$$

(4.1)

where $k_1 k_2$ are amplitude parameters and $f_F$ is set to the natural frequency of the bridge. The parameters $k_1 k_2$ are defined as:

$$k_1 = \sqrt{0.1BL}$$

(4.2)

Where $B$ is the width of the deck and $L$ is the length of the bridge but limited by the length of 5 spans.

$$k_2 = 150 \quad \text{if} \quad f_F \leq 2.5Hz$$

(4.3)

$$k_2 = \frac{125}{f_F} \quad \text{if} \quad 2.5Hz < f_F < 3.5Hz$$

(4.4)

The document also provides accepted values on the damping in terms of critical damping ratio $\zeta$

- $\zeta = 0.005$ for steel structures
- $\zeta = 0.006$ for wood-, concrete and composite structures.

The document provides simplified methods for hand calculations but also allows the use of computer programs such as FE-softwares. No further guidelines, limits or consideration to lateral vibrations are presented in this document.

With the introduction of Eurocode the document Bro 2004 has been replaced by the document TRVK Bro 11 which in turn provides references to Eurocode. Therefore this methods is not included any further in this report.
4.1. CURRENT GUIDELINES

4.1.2 Eurocode

EC0
Recommendations regarding comfort in terms of maximum acceleration can be found in the appendix of EC0 (A2.4.3.2) for pedestrians in the serviceability limit state [10]. Vertical and horizontal acceleration limits are presented in table 4.1 and recommended if the natural frequencies of a footbridge are less than 5 Hz for vertical modes and less then 2.5 Hz for lateral or torsional modes.

<table>
<thead>
<tr>
<th>Case</th>
<th>Limit $[m/s^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical vibrations</td>
<td>0.7</td>
</tr>
<tr>
<td>Horizontal vibrations during normal use</td>
<td>0.2</td>
</tr>
<tr>
<td>Horizontal vibrations during exceptional use</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The recommendations for vertical vibrations are in the same range as other proposed limits, although the approach of only considering one limit regardless of design situation may result in too strict requirements for footbridges rarely subjected to large crowds. Furthermore, the horizontal recommendations for normal and exceptional use exceeds the limit for lateral synchronization proposed in the method by Sêtra. This is discussed further on in this report.

EC1-2
According to EC1-2 5.7 the dynamic effects from pedestrian loading on footbridges should be considered during the design phase. It presents a few pedestrian activities with the corresponding frequency ranges at which they occur, see figure 4.2.

<table>
<thead>
<tr>
<th>Case</th>
<th>Frequency range $[Hz]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking vertical</td>
<td>1.0 - 3.0</td>
</tr>
<tr>
<td>Walking lateral</td>
<td>0.5 - 1.5</td>
</tr>
<tr>
<td>Running vertical</td>
<td>3.0</td>
</tr>
</tbody>
</table>

No dynamic load model is presented but left to the designer.

4.1.3 BS 5400
The guidelines provided by the British Standard Institution BS 5400 are one of the earliest to include dynamic effects due to pedestrian loading [7]. These guidelines are to a great extent based on work by Blanchard et al. on the subject and include models such as the walking force model described in section 3.3.2 and a frequency dependant RMS acceleration limit described in section 2.4.2, Eq. 2.63. Initially only vertical vibrations were required to be analysed but after the the opening of the Millennium bridge in 2000 where large lateral vibrations were observed an updated version was published that required the lateral vibrations to be included in the analysis as well.
4.2 Finite Element Models

Since a modern footbridge more often than not have a rather complex structure a Finite Element (FE) software is commonly used. It is an efficient tool for calculating stress distributions and deflections as well as determining dynamic behaviour. This gives the designer the possibility to design much more complex structures compared to what simplified methods and hand calculations would allow. The accuracy of a FE-model depends completely on the assumptions the model is based on. It is therefore crucial that the model resembles the actual behaviour as closely as possible.

The usage of FE-software in this report focuses mainly on two steps. The first is the eigenvalue analysis applied as a frequency step and used to determine the eigenmodes \( \phi \), corresponding natural frequencies \( f_n \) and generalized mass \( M_n \). The second is the modal analysis used to compute the response from dynamic loading.

4.2.1 Stiffness

One of the most important parameters governing the natural frequencies of a structure is the stiffness \( K \). In order to represent the stiffness accurately the model has to be as realistic as possible and the elements chosen depending on the object currently modelling.

Due to poor choice of element types or size the stiffness might easily be misrepresented in the model. This in turn will provide inaccurate natural frequencies. In order to illustrate this a simply supported plate is modelled in the FE-software Abaqus using both 3D solid elements with various mesh sizes and 2D shell elements, see figure 4.1. By applying a frequency step in Abaqus the natural frequencies are obtained and compared. The element types used in this comparison are:

- 3D solid linear elements (8 nodes)
- 3D solid quadratic elements (20 nodes)
- 2D Shell elements

![Figure 4.1: Sketch of modelled plate with the thickness 0.01 m](image)

As a reference hand calculations are performed on this plate, the natural frequencies are obtained using the well known equation [3]:

\[
f_{ref,n} = \frac{n^2 \pi}{2L^2} \sqrt{\frac{EI}{\rho S}}
\]  

(4.5)

where \( n \) is the modal number, \( L \) is the length of the span, \( E \) is the Young’s modulus, \( I \) is the moment of inertia for the current mode, \( \rho S \) is the linear density of the plate. For geometrical parameters, see figure 4.1. The material parameters used are \( E_{steel} = 210 \text{MPa} \) and \( \rho_{steel} = 7850 \text{kg/m}^2 \).
4.2. **FINITE ELEMENT MODELS**

\[
f_{ref,1} = \frac{1^2 \cdot \pi}{2 \cdot 2^2} \sqrt{\frac{210 \cdot 10^9 \cdot 0.6 \cdot 0.01}{7850 \cdot 0.6 \cdot 0.01^3}} = 5.8633\,Hz \tag{4.6}
\]

\[
f_{ref,2} = \frac{2^2 \cdot \pi}{2 \cdot 2^2} \sqrt{\frac{210 \cdot 10^9 \cdot 0.6 \cdot 0.01^3}{7850 \cdot 0.6 \cdot 0.01}} = 23.4533\,Hz \tag{4.7}
\]

These two natural frequencies will serve as a reference for the comparison of element settings in the FE-analysis.

**Mesh 1**
Linear 8 node solid elements with a size of 0.01 m, see figure 4.2.

![Figure 4.2: Mesh 1 - Mesh and first two eigenmodes with the natural frequencies at \(f_1 = 0.61150\,Hz\) and \(f_2 = 2.4454\,Hz\)](image)

**Mesh 2**
Linear 8 node solid elements with a mesh size of 0.005 m, see figure 4.3.

![Figure 4.3: Mesh 2 - Mesh and first two eigenmodes with the natural frequencies at \(f_1 = 5.1101\,Hz\) and \(f_2 = 20.6110\,Hz\)](image)

**Mesh 3**
Linear 8 node solid elements with a mesh size of 0.01 m and \(t = 0.002\) m, see figure 4.4.

![Figure 4.4: Mesh 3 - Mesh and first two eigenmodes with the natural frequencies at \(f_1 = 5.7756\,Hz\) and \(f_2 = 23.3160\,Hz\)](image)

**Mesh 4**
Quadratic 20 node solid elements with a mesh size of 0.01 m, see figure 4.5.

![Figure 4.5: Mesh 4 - Mesh and first two eigenmodes with the natural frequencies at \(f_1 = 5.8892\,Hz\) and \(f_2 = 23.7550\,Hz\)](image)

**Mesh 5**
Linear 4 node shell elements with a mesh size of 0.01 m, see figure 4.6.

![Figure 4.6: Mesh 5 - Mesh and first two eigenmodes with the natural frequencies at \(f_1 = 5.8890\,Hz\) and \(f_2 = 23.7570\,Hz\)](image)
CHAPTER 4. METHODS

4.2. FINITE ELEMENT MODELS

Comparison

The natural frequencies computed for Mesh 1 - 5 are compared to the natural frequencies calculated in Eq. 4.6 and Eq. 4.7, see table 4.3.

Table 4.3: Comparison of natural frequencies for varying mesh options

<table>
<thead>
<tr>
<th>Method</th>
<th>$f_1 [Hz]$</th>
<th>$f_2 [Hz]$</th>
<th>$\Delta_1$</th>
<th>$\Delta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand calculations</td>
<td>5.8633</td>
<td>23.4533</td>
<td>0 %</td>
<td>0 %</td>
</tr>
<tr>
<td>FE Mesh 1</td>
<td>0.6115</td>
<td>2.4454</td>
<td>858.84 %</td>
<td>859.08 %</td>
</tr>
<tr>
<td>FE Mesh 2</td>
<td>5.1101</td>
<td>20.6110</td>
<td>14.74 %</td>
<td>13.79 %</td>
</tr>
<tr>
<td>FE Mesh 3</td>
<td>5.7756</td>
<td>23.3160</td>
<td>1.52 %</td>
<td>0.59 %</td>
</tr>
<tr>
<td>FE Mesh 4</td>
<td>5.8892</td>
<td>23.7550</td>
<td>-0.44 %</td>
<td>-1.27 %</td>
</tr>
<tr>
<td>FE Mesh 5</td>
<td>5.8890</td>
<td>23.7570</td>
<td>-0.44 %</td>
<td>-1.28 %</td>
</tr>
</tbody>
</table>

where

$$\Delta_n = \frac{f_n - f_{ref,n}}{f_n} \quad (4.8)$$

Conclusions

It is evident from this example that the choice of element types and size has a significant influence on the outcome of the simulation. The reason for the extreme discrepancy acquired from Mesh 1 is because a single linear element was intended to capture the bending. This is a common problem when using linear elements where the shape function describing the geometry of the element is linear. Therefore the element can only bend through sheer deformation, see figure 4.7 and 4.8 for comparison between expected behaviour and FE-behaviour behaviour. This is called Sheer locking. By increasing the number of elements, each element is subjected to smaller bending and therefore the response is captured more accurately. The results in table 4.3 indeed seem to converge by increasing the number of elements capturing the bending.

By instead using quadratic elements the behaviour is captured with more nodes, hence a quadratic description of the deformed element can be made. This allows the element to capture bending in a more accurate manner. The results from Mesh 4 seem to confirm this where the natural frequency was captured accurately with a single element.

The last configuration made use of shell elements, these elements have no thickness and are based on the assumption of plane stress and are therefore only applicable when the thickness is small in comparison to the width and length. The absence of a thickness means that bending can be captured without risk of sheer locking. This makes shell elements far superior in these situations.

For a more in depth explanation of elements and their properties the reader is referred to literature on this subject [13], this report does not intend to cover the theory of FE-modelling, only provide examples of problems associated with the Finite Element Method.
4.2. FINITE ELEMENT MODELS

4.2.2 Generalized mass

The generalized mass of the structure determines both the natural frequencies, see Eq. 2.10, and the response in terms of accelerations, see Eq. 2.40. It should therefore be given special considerations during the design of the footbridge. The static weight of pedestrians on the footbridge contribute to the total mass of the structure and should be accounted for both when determining the natural frequencies and during the dynamic analysis of the footbridge. Due to its impact on natural frequencies and accelerations it can be a useful tool when making adjustments to the design in order to limit the response or adjust the natural frequencies.

The simplified relation for approximating the generalized mass based on a sinusoidal modeshape in Eq. 2.47 is applied to the plate used in section 4.2.1. The shell element model (Mesh 5) provides the following vertical modes, see figure 4.9.

The generalized mass for the three modes in figure 4.9 is calculated by hand

\[ \hat{m} = \bar{m}L^2 = 7850 \cdot 0.01 \cdot 0.6 \cdot L = 47.10 \text{ kg} \]  \hspace{1cm} (4.9)

The computed generalized mass \( M_n \) is compared to the calculated generalized mass in Eq. 4.9, see table 4.4.

<table>
<thead>
<tr>
<th>Method</th>
<th>( M_n )</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Mode 1</td>
<td>45.30</td>
<td>-4.0 %</td>
</tr>
<tr>
<td>Vertical Mode 2</td>
<td>41.97</td>
<td>-12.2 %</td>
</tr>
<tr>
<td>Vertical Mode 3</td>
<td>39.21</td>
<td>-20.1 %</td>
</tr>
</tbody>
</table>

where

\[ \Delta_n = \frac{M_n - \hat{m}}{M_n} \] \hspace{1cm} (4.10)

Based on table 4.4 it is clear that Eq. 2.47 provides a fair approximation of the generalized mass for the first mode. It tends to overestimate the generalized mass for higher modes due to the simplification of sinusoidal modeshape. However, the approximation is on the safe side and can be considered a viable option for hand calculation or verification of FE-results.
4.2.3 Damping

The damping used in a FE-model is dependant on several factors, the most common one being frequency as described in this report. As this is a crucial part when determining the response due to harmonic loading it is important to not overestimate this property. The damping of a structure is also dependant on the magnitude of the deformations, damping used for the serviceability limit state are significantly lower compared to the damping used in earthquake analysis. Damping parameters are usually provided in design codes and guidelines as design values, see figure 2.9.

In practice the pedestrians on the footbridge will absorb some vibrations in legs and joints, this results in a slightly increased overall damping. This is usually not accounted for during the design process but can be considered beneficial in limiting the response.

4.2.4 Dynamic response

The dynamic load is as mentioned previously commonly described as the deterministic load model, see section 3.1. Due to the uncoupling of modes in a modal expansion, see section 2.2.1, each mode and harmonic in the deterministic load model can be applied and computed individually. The total response is a linear combination of each individual response.

The simple plate model used in section 4.2.1 is subjected to a distributed sinusoidal load $q$ of $1N/m^2$ applied as a pressure in the vertical direction, see figure 4.10. Furthermore, the critical damping ratio is set to $\xi = 0.4\%$.

\begin{equation}
q = 1.0 \cos(2\pi f_n t)
\end{equation}

where $f_n$ is the natural frequency of the plate for the current mode and $t$ is the time. The first natural mode is considered and the load is placed in such a way that this mode is excited, see section 4.2.1 for modeshapes.

![Figure 4.10: Applied distributed load q from Eq. 4.11](image)

The response is plotted for a node at the centre of the plate and presented in figure 4.11.

![Figure 4.11: Acceleration response for plate, maximum peak accelerations $a_{max} = 2.0m/s^2$.](image)
4.2.5 Verification of results

When using a FE-model it is convenient to verify the results using hand calculations. The results from section 4.2.4 are verified using Eq. 2.58 where the generalized force $P_1$ and mass $M_1$ are determined through Eq. 2.54 for the first natural mode.

$$\ddot{u}_{\text{max},1,n} = -\mu_{1,n} \frac{P_1 M_1}{2 \xi_1}$$  \hspace{1cm} (4.12)

where $\xi_1$ is the critical damping ratio for the first mode and $\mu_{1,n}$ is the modal value for the current node $n$.

For a purely vertical mode the maximum modal value in the vibrating direction is $\mu_{1,max} = 1.0$. The generalized force is determined based on the eigenvector $\phi$ of the plate for the current mode.

$$P_1 = \phi^T p(t) = 0.7317N$$  \hspace{1cm} (4.13)

where $p(t)$ is the load vector used in section 4.2.4 and presented in figure 4.10.

This can be compared to the simplified case based on sinusoidal response in Eq. 2.49.

$$\hat{p} = \frac{2}{\pi} \cdot p(t) = \frac{2}{\pi} \cdot A \cdot q(t) = 0.7639N$$  \hspace{1cm} (4.14)

where the area of the plate $A = 2.0 \cdot 0.6 = 1.0m^2$.

Applying Eq. 4.12 with the computed generalized mass in section 4.2.2 and the generalized force in Eq. 4.13.

$$\ddot{u}_{\text{max}} = 1.0 \cdot \frac{0.7317}{45.30 \cdot 2 \cdot 0.004} = 2.0m/s^2$$  \hspace{1cm} (4.15)

This is the same response as computed in the FE-model shown in figure 4.11.

4.2.6 Boundary conditions

One of the most difficult assumptions to make in the design process of a footbridge are the boundary conditions. The idealized behaviour of boundary conditions in a FE-software rarely captures the actual behaviour of bearings due to imperfections, friction and loss of functionality over time. No general method for modelling bearings in the serviceability limit state exists but is left to the designer for each individual structure. It is therefore important to consider the effects of different assumptions when creating a FE-model.

In the report "European design guide for footbridge vibration" from the conferance Footbridge 2008 the behaviour of joints in footbridges is described to depend on the subjected deformations [5]. A connection defined as hinged in the ultimate limit state, see figure 4.12, when subjected to small deformations, serviceability limit state, might be more accurately modelled as a rigid connection, see figure 4.13. This in turn changes system properties such as eigenmodes, it is therefore important to consider the deformation and adapt the model to the expected deformations.

![Figure 4.12: Modeshape - ULS](image)

![Figure 4.13: Modeshape - SLS](image)

The modeshape in figure 4.13 can be considered through the use of an effective length $L_{\text{eff}} < L_{\text{tot}}$. 

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CHAPTER 4. METHODS

4.2. FINITE ELEMENT MODELS

This reduction in length results in an increased stiffness which in turn increases the natural frequencies. However, there is currently no guidelines on the dynamic behaviour during small deformations to implement this in the design process.

The boundary conditions for the case Munktell are presented in section 1.3.1. The supports are modelled on springs to account for the soil stiffness. The spring stiffness used in the model is determined based on the initial settlements due to the self weight as described in TRVR Bro 11 and is common practice in bridge engineering. However, this might not accurately capture the dynamic stiffness since both the time period and the magnitude of the pedestrian loads used differ substantially to the self weight [5].

This is currently researched for the purpose of modelling bridges intended for rail road use, therefore the guidelines are still scarce on the subject. However, for short term loading and the significantly smaller loads, the settlements of the soil can be assumed smaller compared to the settlements calculated based on recommendations by TRVR Bro 11. Therefore the soil stiffness commonly used in FE-models might underestimate the dynamic stiffness. In order to illustrate this the natural frequencies for case Munktell are computed in Abaqus based on two different assumptions. First the spring stiffness determined through TRVR Bro 11 is used, see figure 4.14. In order to study the opposite extreme where the stiffness is assumed infinite, the boundary conditions are modelled as fixed, see figure 4.15.

![Figure 4.14: Munktell - Spring supports](image)

![Figure 4.15: Munktell - Fixed supports](image)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f_n [Hz]$ Spring supports</th>
<th>$f_n [Hz]$ Fixed supports</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0153</td>
<td>3.0168</td>
</tr>
<tr>
<td>2</td>
<td>2.5381</td>
<td>3.2579</td>
</tr>
<tr>
<td>3</td>
<td>3.2606</td>
<td>3.8010</td>
</tr>
<tr>
<td>4</td>
<td>4.0485</td>
<td>4.7304</td>
</tr>
<tr>
<td>5</td>
<td>4.6535</td>
<td>5.3965</td>
</tr>
</tbody>
</table>

From table 4.5 it is clear that the increased stiffness from the fixed supports results in higher natural frequencies. However, since limited guidelines exist on the subject a conservative approach might be to use the spring supports recommended in TRVR Bro 11. In practice the stiffness might be significantly higher and is in this case considered beneficial. For natural frequencies in the low end of the natural walking frequency range the increased stiffness might result in a less beneficial outcome.
4.2.7 Time - Stepping

When using a time-stepping procedure in a FE-software a time interval and time increment is specified by the designer. The FE-software solves the equation of motion for each time step, it therefore important to specify time steps small enough to capture the behaviour properly. If the step size is too big a misrepresentation of the behaviour might occur. This commonly results in an underestimation of the true response, see figure 4.16.

![Figure 4.16: Example of poorly captured response](image)

This is illustrated by yet another example using the simple plate modelled as shell elements. The plate is subjected to the load in section 4.2.4, see figure 4.17 and 4.18 for a comparison of response using different time increments.

![Figure 4.17: Response with time increment \( t_{\text{incr}} = 0.05 \text{s} \), \( a_{\text{max}} \approx 1.5 \text{m/s}^2 \).](image)

![Figure 4.18: Response with time increment \( t_{\text{incr}} = 0.005 \text{s} \), \( a_{\text{max}} \approx 2.0 \text{m/s}^2 \).](image)

From this it is clear that the increment size should be considered thoroughly, the size clearly has a significant impact on the results. As a guideline the following relation is commonly used [15]

\[
\Delta t = \frac{1}{10 \cdot f_n} \quad (4.16)
\]

where \( \Delta t \) is the time increment and \( f_n \) is the natural frequency intended to capture.

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4.3 Generalized SDOF method

This method for calculating footfall induced vibration is partially based on a method developed by M.R. Willford and P. Young that unlike other methods relies on a more analytical approach of structural dynamics where complex MDOF-systems are reduced to generalized SDOF-systems. It is therefore suitable for calculating vibrations due to walking on any surface, including bridges [2]. However, the method as described in the guiding document does not consider lateral vibrations or different crowd formations and is therefore in this report developed further based on current knowledge.

4.3.1 Calculations

The following is a summary of the method developed by Willford & Young as presented in the document "A design guide for footfall induced vibrations" as a publication by the Cement and Concrete Industry [2].

The method assumes a walking frequency \( f_w \) in the range of 1 - 2.8 Hz and the presence of the first four harmonics \( f_h \) of the walking frequency:

\[
f_h = h f_w
\]

where \( h = (1,2,3,4) \)

The dynamic load factor, or DLF, is the ratio of the dynamic load and the total load of a pedestrian, 700 N. For this method the vertical DLF is taken from extensive measurements by S. Kerr, see section 3.1.2 for Dynamic load factors. Through statistical analysis a mean value with regard to walking frequency and the corresponding deviation is calculated. The design value in this method is set as the 75 % fractile, see table 4.6.

<table>
<thead>
<tr>
<th>Harmonic number</th>
<th>Harmonic frequency range ( f_h ) [Hz]</th>
<th>DLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0 - 2.8</td>
<td>0.37 ((f - 0.95) \leq 0.56)</td>
</tr>
<tr>
<td>2</td>
<td>2.0 - 5.6</td>
<td>0.054 + 0.0044(f)</td>
</tr>
<tr>
<td>3</td>
<td>3.0 - 8.4</td>
<td>0.026 + 0.005(f)</td>
</tr>
<tr>
<td>4</td>
<td>4.0 - 11.2</td>
<td>0.010 + 0.0051(f)</td>
</tr>
</tbody>
</table>

The eigenmodes included in the analysis are determined by their corresponding frequency. Since the maximum frequency of interest is 11.2 Hz (4th harmonic of 2.8 Hz) all modes with a corresponding frequency in the range 1.0 - 11.2 Hz should be considered.

The maximum peak accelerations are calculated for each mode as a real and imaginary part for the current harmonic frequency \( f_h \) and mode \( m \).

\[
a_{\text{real},h,m} = \left( \frac{f_h}{f_m} \right)^2 \frac{F_h \mu_{r,m} \mu_{e,m} P_{h,m}}{m_m A_m} \frac{A_m}{A_m^2 + B_m^2} \tag{4.18}
\]

\[
a_{\text{imag},h,m} = \left( \frac{f_h}{f_m} \right)^2 \frac{F_h \mu_{r,m} \mu_{e,m} P_{h,m}}{m_m B_m} \frac{B_m}{A_m^2 + B_m^2} \tag{4.19}
\]

where \( F_h \) is a harmonic point load, \( \mu_{e,m} \) is the mode shape value at the point of excitation, \( \mu_{r,m} \) is the mode shape value at the point where the response is measured, \( m_m \) is the modal mass and \( f_m \) is the natural frequency for the current mode \( m \).
4.3. GENERALIZED SDOF METHOD

Furthermore:

\[ A_m = 1 - \left( \frac{f_h}{f_m} \right)^2 \]  
(4.20)

\[ B_m = 2 \xi_m \frac{f_h}{f_m} \]  
(4.21)

and

\[ \rho_{h,m} = 1 - e^{-2\pi \xi_m N_h} \]  
(4.22)

\[ N_h = 0.55 \cdot h \frac{L}{I} \]  
(4.23)

where \( \xi_m \) is the critical damping ratio for the current mode \( m \), \( L \) the length of the span and \( I \) the stride length of the pedestrian.

The reduction factor \( \rho_{h,m} \) is introduced to account for the time it takes a pedestrian to cross the span and is determined for each mode and harmonic. Because the accelerations for a damped system increases with time towards a maximum value during harmonic loading a single pedestrian will not be able to reach the maximum peak accelerations if the span is short. Therefore Eq. 4.22 limits the maximum peak accelerations for shorter spans.

The real and imaginary part of the accelerations are summed for all included modes:

\[ a_{\text{real},h} = \sum_{m=1}^{M} a_{\text{real},h,m} \]  
(4.24)

\[ a_{\text{imag},h} = \sum_{m=1}^{M} a_{\text{imag},h,m} \]  
(4.25)

The magnitude of the acceleration for the current harmonic force is determined:

\[ |a_h| = \sqrt{a_{\text{real},h}^2 + a_{\text{imag},h}^2} \]  
(4.26)

The accelerations are evaluated as a response factor \( R_h \) for the current harmonic.

\[ R_h = \frac{|a_h|}{a_R} \]  
(4.27)

where \( a_R \) is a baseline value, the limit of perceived peak accelerations for a person at rest. The limit is frequency dependant as seen in figure 2.15 and table 4.7.

<table>
<thead>
<tr>
<th>Harmonic frequency ( f_h ) [Hz]</th>
<th>Baseline peak acceleration ([m/s^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_h &lt; 4.0 )</td>
<td>( a_R = 0.0141 / \sqrt{f_h} )</td>
</tr>
<tr>
<td>4.0 &lt; ( f_h &lt; 8.0 )</td>
<td>( a_R = 0.0071 )</td>
</tr>
<tr>
<td>( f_h &gt; 8.0 )</td>
<td>( a_R = 2.82\pi f_h \cdot 10^{-4} )</td>
</tr>
</tbody>
</table>

The total response factor \( R \) for the current forcing frequency \( f_w \) is the square root sum of the four harmonic response factors \( R_h \):

\[ R = \sqrt{R_1^2 + R_2^2 + R_3^2 + R_4^2} \]  
(4.28)

This procedure is repeated for forcing frequencies within the range 1.0 - 2.8 Hz.

In the guiding document a limit of \( R = 60 \) is recommended based on perceived comfort, however, this does not account for lateral synchronization effects described in section 3.2.2.
CHAPTER 4. METHODS  4.3. GENERALIZED SDOF METHOD

Summary
Due to the large number of calculations needed the method is suitable for scripting using a numerical software such as MATLAB or Python. In order to clarify the steps a short summary is presented without the equations:

- For a walking frequency $f_w$ determine the four lowest harmonic frequencies $f_h$.
- For a given harmonic frequency determine the accelerations for each mode within the range 1.0 - 11.2 Hz.
- Sum the accelerations for each mode within the current harmonic frequency. Each harmonic of the current forcing frequency results in a response factor $R_h$.
- The total response factor for a forcing frequency $f_w$ is the square root sum of the harmonic response factors.
- Repeat the calculations for forcing frequencies $f_w$ within the range 1.0 - 2.8 Hz. The result is conveniently illustrated by plotting the response against the forcing frequency $f_w$.

Clarification of procedure
The accelerations are determined through Eq. 4.18 and 4.19. As described section 2.2.1 the generalized force $P_m$ for a mode $m$ is written as

$$ P_m(t) = \phi_m^T \Phi(t) $$

(4.29)

For the case of a concentrated force only the node of excitation $n$ contains a value not equal to zero. Eq. 4.29 becomes

$$ P_m(t) = \mu_{m,n} \cdot \Phi(t) $$

(4.30)

where $\mu_{m,n}$ is a modal value for the node $n$ in the eigenvector $\phi_m$. This corresponds to the terms $F_h \mu_{e,m}$ in Eq. 4.18 and 4.19.

For this report the case of an evenly distributed load from crowd loading is analysed. This can be expressed as a generalized force based on the modeshape using Eq. 2.45 or Eq. 2.53.

Eq 4.18 and 4.19 are rewritten using the generalized force $P_m$ and mass $M_m$ for a mode $m$ and harmonic $h$.

$$ a_{\text{real},h,m} = \left( \frac{f_h}{f_m} \right)^2 \frac{P_m \mu_{r,m} \rho_{h,m} A_m}{M_m} \frac{A_m}{A_m^2 + B_m^2} $$

(4.31)

$$ a_{\text{imag},h,m} = \left( \frac{f_h}{f_m} \right)^2 \frac{P_m \mu_{r,m} \rho_{h,m} B_m}{M_m} \frac{B_m}{A_m^2 + B_m^2} $$

(4.32)

This is a more convenient form due to the clear use of generalized quantities.

Simplified calculation during resonance
Eq. 4.31 - 4.32 can be simplified for the case of $f_h = f_m$.

$$ a_{\text{peak},h,m} = a_{\text{real},h,m} + a_{\text{imag},h,m} = \frac{P_m \mu_{r,m} \rho_{h,m} A_m + B_m}{M_m} \frac{A_m}{A_m^2 + B_m^2} $$

(4.33)

where

$$ A = 0 \quad B = 2 \xi $$

(4.34)

Eq. 4.33 becomes

$$ a_{\text{peak},h,m} = \rho_{h,m} \cdot \frac{P_m}{M_m} \frac{1}{2 \xi_m} \mu_{r,m} $$

(4.35)

This is the same expression as derived in Eq. 2.58 for the case of forced vibrations of a damped system, except for the introduced reduction factor $\rho_{h,m}$.
4.3. GENERALIZED SDOF METHOD

4.3.2 Load Model

The calculations based on the guiding document by Willford & Young and described in section 4.3.1 is based on a single pedestrian in complete synchronization with the bridge simulated as a point load. This provides limited possibilities to analyse different loading situations based on predicted traffic. For this report is of interest to further develop this method based on the theory described in section 2 and 3 of this report in order to increase its applicability as a design tool.

In order to account for groups and crowds of pedestrians it is necessary to consider the number of pedestrians in synchronization with the bridge and each other. For this the method described and implemented by Sétra of considering only an equivalent number of pedestrians for the dynamic load is implemented here as well, see section 3.2. Furthermore, no account for different loading situations is proposed in the guiding document, therefore the traffic classes described by Sétra of sparse crowd, dense crowd and very dense crowd are implemented where the corresponding pedestrian densities are 0.5, 0.8 and 1.0 pedestrians/m².

Several dynamic load factors have been presented throughout this report, see section 3.1.3 and 3.1.4. The dynamic load factors presented in the guiding document are frequency dependant factors proposed from measurements by S. Kerr [2]. It is of interest to analyse other load factors as well. Therefore the dynamic load factors are treated simply as variables, during the case studies the choice of DLF is studied as well.

Furthermore, no account is made to lateral vibrations in the guiding document, however, due to the simplicity of the method it is possible to account for lateral vibrations by selecting the parameters such as generalized mass and force for the lateral direction and making use of lateral dynamic load factors, see section 3.1.4.

4.3.3 Implementation

To implement this method the following data is required for each mode \( m \) of the footbridge

- Natural frequency \( f_n \)
- Generalized mass \( M_m \)
- Modeshape \( \phi_m \) containing the modal value \( \mu_{m,n} \) for the node \( n \)
- Critical damping ratio \( \xi_m \)

As described in previous section the number of pedestrians is determined based on findings by Sétra and the dynamic load factors are treated as variables in order to analyse how the choice of load model influence the response. The implementation is presented in the form of a manual in appendix A of this report.
4.4 Sétra method

The method developed by the Technical Department of Transport, Roads and Bridge Engineering Sétra is presented as a Technical guide, originally published in French and later translated to English. The proposed guidelines rely both on current knowledge and experimental results from tests performed on the footbridge Solferino as well as on a testing platform, see figure 4.19 and 4.20 [3]. The method provides a complete methodology for determining the dynamic response of a pedestrian bridge during service limit state, including classification of comfort requirements, dynamic load cases and bridge classes.

4.4.1 Definitions and Classifications

Footbridge class
Firstly, the predicted traffic loads are determined, this is defined as footbridge classes I - IV. It is reasonable to assume that a footbridge in an urban area will be subjected to higher loads compared to a footbridge in a sparsely populated area. The classes described by Sétra are:

- Class I: Urban areas, high pedestrian density, regular crowd loading.
- Class II: Urban areas, occasional crowd loading.
- Class III: Standard footbridge use, occasional crossing by large groups.
- Class IV: Seldom used footbridge, sparsely populated areas, calculations are not required.

Since footbridge class IV does not require a dynamic analysis it is recommended to avoid this class due to the recommendations in section 4.1.2.

Comfort levels
In order to specify the requirements on the footbridge the method suggests vertical and lateral peak acceleration ranges with regard to perceived comfort by a pedestrian, see table 4.8.
4.4. SÉTRA METHOD

### Table 4.8: Vertical peak acceleration ranges [3]

<table>
<thead>
<tr>
<th>Range</th>
<th>Peak acceleration $[m/s^2]$</th>
<th>Comfort</th>
<th>Perceived by pedestrians</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0 - 0.5</td>
<td>Maximum</td>
<td>Not perceived</td>
</tr>
<tr>
<td>2</td>
<td>0.5 - 1.0</td>
<td>Average</td>
<td>Merely perceived</td>
</tr>
<tr>
<td>3</td>
<td>1.0 - 2.5</td>
<td>Minimum</td>
<td>Perceived but not intolerable</td>
</tr>
<tr>
<td>4</td>
<td>$&gt; 2.5$</td>
<td>Unacceptable</td>
<td>Not accepted</td>
</tr>
</tbody>
</table>

Horizontal peak acceleration ranges with regard to perceived comfort are presented in a similar manner, however, due to “lock-in” effects at peak accelerations $> 0.10 \text{ m/s}^2$, it is recommended to not exceed this value, see chapter 3.2.2.

### Natural frequency range

In order to determine the sensitivity to dynamic loading the natural frequencies for the footbridge needs to be determined. This can be performed using hand calculations, or more commonly, a FE-software. Two assumptions regarding the mass from pedestrians are considered. The first assumption is an empty footbridge and the second a loaded footbridge with a pedestrian density of 1 pedestrian/$m^2$. The natural frequencies should be calculated based on both assumptions. The closer the natural frequencies are to the pedestrian walking frequency the more likely it is for resonance to occur. The less favourable situation should be considered. The suggested frequency ranges used to assess the likelihood of resonance are presented in table 4.9.

### Table 4.9: Natural frequency ranges [3]

<table>
<thead>
<tr>
<th>Range</th>
<th>Harmonic</th>
<th>$f_n [Hz]$ Vertical</th>
<th>$f_n [Hz]$ Lateral</th>
<th>Risk of resonance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1st</td>
<td>1.7 - 2.1</td>
<td>0.5 - 1.1</td>
<td>Maximum</td>
</tr>
<tr>
<td>2</td>
<td>1st</td>
<td>1.0 - 1.7 and 2.1 - 2.6</td>
<td>0.3 - 0.5 and 1.1 - 1.3</td>
<td>Medium</td>
</tr>
<tr>
<td>3</td>
<td>2nd</td>
<td>2.6 - 5.0</td>
<td>1.3 - 2.5</td>
<td>Low</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>$&gt; 5.0$</td>
<td>$&gt; 2.5$</td>
<td>Negligible</td>
</tr>
</tbody>
</table>

### Required calculations

Based on the defined footbridge class and natural frequency range the required calculations in terms of load cases are determined according to figure 4.21. For example a footbridge in class II and a natural frequency within the range 2 should be subjected to the load described in load case 1. The load model and a description of the corresponding load cases is described in the next section.

![Load cases to select for acceleration checks](image)

**Figure 4.21:**

Required calculations according to the method by Sétra [3]
4.4.2 Load Model

The load model used is the deterministic load model as described in section 3.1. Crowd load is defined as an evenly distributed load over the entire walkway of the bridge and determined by a crowd density measured in pedestrians per square meter. The equivalent number of pedestrians \(N_{eq}\) is the number of pedestrians in synchronization with each other and the footbridge, see section 3.2. Vertical interaction effects are accounted for by considering a different relation describing the equivalent number of pedestrians at densities above 1 pedestrian/m², see section 3.2.1.

As mentioned in section 3.4.1 the loads are defined using load cases. The load cases used in this method are defined in table 4.10.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sparse and dense crowd</td>
</tr>
<tr>
<td>2</td>
<td>Very dense crowd</td>
</tr>
<tr>
<td>3</td>
<td>Crowd compliment - 2nd harmonic</td>
</tr>
</tbody>
</table>

To account for the probability of pedestrians walking in the same frequency as the natural frequency of the bridge a reduction factor \(\psi\) is introduced, see figure 4.22 and 4.23. It is clear that in a crowd of people it is unlikely that enough people would walk in a frequency > 2.6 Hz by chance to be of any practical concern.

![Figure 4.22: Vertical and longitudinal reduction factor \(\psi\).](image)

![Figure 4.23: Lateral reduction factor \(\psi\).](image)

Case 1

Effects of the first harmonic without vertical interaction effects. The corresponding pedestrian densities for Case 1 are presented in table 4.11 and load models in table 4.12.

<table>
<thead>
<tr>
<th>Footbridge Class</th>
<th>Crowd</th>
<th>(d) [pedestrians/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>Sparse Crowd</td>
<td>0.5</td>
</tr>
<tr>
<td>II</td>
<td>Dense crowd</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Direction</th>
<th>(q) [N/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>(d \cdot 280 \cos(2\pi f_{nt} t) \cdot 10.8 \sqrt{\frac{\zeta}{N}} \cdot \psi)</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>(d \cdot 140 \cos(2\pi f_{nt} t) \cdot 10.8 \sqrt{\frac{\zeta}{N}} \cdot \psi)</td>
</tr>
<tr>
<td>Lateral</td>
<td>(d \cdot 35 \cos(2\pi f_{nt} t) \cdot 10.8 \sqrt{\frac{\zeta}{N}} \cdot \psi)</td>
</tr>
</tbody>
</table>
The corresponding dynamic load factors for the 1st harmonic are 0.4 for vertical loads, 0.2 for longitudinal loads and 0.05 for lateral loads. This can be compared to other load models in section 3.1.3, section 3.1.4 and section 3.1.5.

**Case 2**
Effects of the first harmonic with vertical interaction effects. The corresponding pedestrian densities are presented in table 4.13 and load models in table 4.14.

<table>
<thead>
<tr>
<th>Footbridge Class</th>
<th>Crowd</th>
<th>(d \ [\text{pedestrians/m}^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Very dense crowd</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 4.14: Load Model - Case 2

<table>
<thead>
<tr>
<th>Direction</th>
<th>(q \ [\text{N/m}^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>(d \cdot 280\cos(2\pi f_w t) \cdot 1.85 \sqrt{1/N} \cdot \psi)</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>(d \cdot 140\cos(2\pi f_w t) \cdot 1.85 \sqrt{1/N} \cdot \psi)</td>
</tr>
<tr>
<td>Lateral</td>
<td>(d \cdot 35\cos(2\pi f_w t) \cdot 1.85 \sqrt{1/N} \cdot \psi)</td>
</tr>
</tbody>
</table>

**Case 3**
Crowd compliment - effects of the second harmonics. The effects of the 2nd harmonic are considered only if the natural frequency is within frequency range 3, see table 4.9. The reduction factor \(\psi\) is shown in figure 4.24 and 4.25.

Case 3 is used for footbridge class I and II, for footbridge class III the effects of the second harmonics are not included. Since interaction effects are considered at pedestrian densities at or above 1 pedestrian/m² the load models provided for this case differ depending on the expected crowd, see table 4.15 for pedestrian densities and table 4.16 for load models.

<table>
<thead>
<tr>
<th>Footbridge Class</th>
<th>Crowd</th>
<th>(d \ [\text{pedestrians/m}^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>Dense crowd</td>
<td>0.8</td>
</tr>
<tr>
<td>I</td>
<td>Very dense crowd</td>
<td>1.0</td>
</tr>
</tbody>
</table>


### Table 4.16: Load Model - Case 3

<table>
<thead>
<tr>
<th>Direction</th>
<th>$q \ [N/m^2]$ Dense crowd</th>
<th>$q \ [N/m^2]$ Very dense crowd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$d \cdot 70 \cos(\omega t) \cdot 10.8 \sqrt{\frac{\zeta}{N}} \cdot \psi$</td>
<td>$d \cdot 70 \cos(\omega t) \cdot 1.85 \sqrt{1/N} \cdot \psi$</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>$d \cdot 35 \cos(\omega t) \cdot 10.8 \sqrt{\frac{\zeta}{N}} \cdot \psi$</td>
<td>$d \cdot 35 \cos(\omega t) \cdot 1.85 \sqrt{1/N} \cdot \psi$</td>
</tr>
<tr>
<td>Lateral</td>
<td>$d \cdot 7 \cos(\omega t) \cdot 10.8 \sqrt{\frac{\zeta}{N}} \cdot \psi$</td>
<td>$d \cdot 7 \cos(\omega t) \cdot 1.85 \sqrt{1/N} \cdot \psi$</td>
</tr>
</tbody>
</table>

The corresponding dynamic load factors for the 2nd harmonic are 0.1 for vertical loads, 0.05 for longitudinal loads and 0.01 for lateral loads. This can be compared to other load models in section 3.1.3, section 3.1.4 and section 3.1.5.

#### 4.4.3 Implementation

The next step is to apply the loads to the bridge. Since the distributed dynamic loads are based on a number of pedestrians completely synchronized with each other and the bridge, both the phase and the angular frequency of the dynamic load has to coincide with the natural frequency and phase of the bridge. This is achieved by applying a distributed load in the direction and frequency as the current eigenmode as shown in figure 4.27 and 4.26. This is usually applied in a FE-software where a dynamic analysis can compute the maximum peak accelerations over time and a comparison can be made to the defined comfort requirements, see figure 4.28.

---

This is repeated for all modes with a natural frequency excited by the load model specified by Sédra in order to ensure that the comfort requirements always are fulfilled.
Chapter 5

Case Studies

5.1 Munktell

5.1.1 Sétra Method - Calculations

The bridge is located in an urban area with high expected traffic, therefore class I is assumed, see section 4.4.1.

Firstly, the natural modes and corresponding frequencies are computed through a frequency step in a FE-software. Two situations are considered, the first one is the loaded structure where a pedestrian density of $1 \text{ pedestrian/m}^2$ is assumed and the second one is the unloaded structure. All natural modes with a frequency $< 5\text{Hz}$ are presented in table 5.1.

Table 5.1: Results from frequency step for loaded and unloaded footbridge

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f_{n1}[\text{Hz}]$</th>
<th>$f_{n2}[\text{Hz}]$</th>
<th>$M_{n1}^\phi[\text{kg}]$</th>
<th>$M_{n1}^\chi[\text{kg}]$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0153</td>
<td>2.0826</td>
<td>173 853</td>
<td>162 866</td>
<td>Lateral</td>
</tr>
<tr>
<td>2</td>
<td>2.5381</td>
<td>2.6259</td>
<td>87 804</td>
<td>82 105</td>
<td>Vertical</td>
</tr>
<tr>
<td>3</td>
<td>3.2606</td>
<td>3.3647</td>
<td>419 680</td>
<td>392 820</td>
<td>Vertical/Longitudinal</td>
</tr>
<tr>
<td>4</td>
<td>4.0485</td>
<td>4.1907</td>
<td>169 806</td>
<td>158 528</td>
<td>Vertical</td>
</tr>
<tr>
<td>5</td>
<td>4.6535</td>
<td>4.8185</td>
<td>84 956</td>
<td>79 486</td>
<td>Vertical</td>
</tr>
</tbody>
</table>

Since the natural frequencies are slightly above the average walking frequency, the case of a fully loaded footbridge $f_{n1}$ results in slightly less beneficial natural frequencies compared to the case of the unloaded footbridge $f_{n2}$. Therefore these natural frequencies are used throughout this case.

The width of the deck $w_d$ is 4.92 m while the effective width $w_{eff}$ is 4.50 m, see section 1.3.1. This can either be accounted for by applying the load to a reduced area or by reducing the magnitude of the load by a factor $w_{eff}/w_d$. For simplicity the later option is used in this case. The effective area of the deck is

$$A_{eff} = 65.5 \cdot 4.5 = 295 \text{m}^2$$ (5.1)

The number of pedestrians on the deck $N$ is defined as

$$N = A_{eff} \cdot d$$ (5.2)

The critical damping ratio is based on the material of the deck. For concrete deck $\xi = 0.6\%$, see figure 2.9.
The model is used as provided by WSP, this means the spring supports as described in TRVR Bro 11 are used. The deformations are assumed to behave as large deformation, see figure 4.12 and 4.13. These assumptions reduce the overall stiffness of the model and can be considered conservative as described in section 4.2.6.

**Mode 1**
The first modeshape is is presented in figure 5.1 and the corresponding calculations in table 5.2.

![Figure 5.1: First natural mode - Lateral](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency ( f_n )</td>
<td>2.0153 Hz</td>
<td>Section 5.1.1 - Table 5.1</td>
</tr>
<tr>
<td>Natural freq. range</td>
<td>Range 3</td>
<td>Section 4.4.1 - Table 4.9</td>
</tr>
<tr>
<td>Load case</td>
<td>Case 3</td>
<td>Section 4.4.1 - Figure 4.21</td>
</tr>
<tr>
<td>Reduction factor ( \psi )</td>
<td>1.0</td>
<td>Section 4.4.2 - Figure 4.25</td>
</tr>
<tr>
<td>Pedestrian density ( d )</td>
<td>1.0</td>
<td>Section 4.4.2 - Case 3</td>
</tr>
</tbody>
</table>

The load is based on load case 3, effects of 2nd harmonics, and for a pedestrian density of 1 pedestrian/m². The applied load is calculated from table 4.16:

\[
q_1 = 1.0 \cdot 7 \cos(2\pi f_w t) \cdot 1.85 \sqrt{1/(A \cdot d)} \cdot \psi \cdot \frac{w_{eff}}{w_d} \tag{5.3}
\]

\[
q_1 = 0.690 \cos(2\pi f_w t) \quad [N/m^2] \tag{5.4}
\]

The load \( q_1 \) from Eq. 5.4 is applied in a lateral direction with the forcing frequency \( f_w = f_n \) as shown in figure 5.2.

![Figure 5.2: Direction and distribution of applied load in order to excite mode 1](image)
The response in 5.4 and 5.3 has the maximum peak acceleration $a_{\text{max}} \approx 0.06\text{m/s}^2$. This satisfies both the recommended limit of $0.1\text{m/s}^2$ to avoid lock-in effects specified by Sétra and the limits recommended by Eurocode for horizontal vibrations during normal use of $0.2\text{m/s}^2$. 
5.1. MUNKTELL

CHAPTER 5. CASE STUDIES

Mode 2
The second modeshape is is presented in figure 5.5 and the corresponding calculations in table 5.3.

![Figure 5.5: Second natural mode - Vertical](image)

Table 5.3: Munktell calculations - Mode 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency (f_n)</td>
<td>2.5381 Hz</td>
<td>Section 5.1.1 - Table 5.1</td>
</tr>
<tr>
<td>Natural freq. range</td>
<td>Range 2</td>
<td>Section 4.4.1 - Table 4.9</td>
</tr>
<tr>
<td>Load case</td>
<td>Case 2</td>
<td>Section 4.4.1 - Figure 4.21</td>
</tr>
<tr>
<td>Reduction factor (\psi)</td>
<td>0.124</td>
<td>Section 4.4.2 - Figure 4.22</td>
</tr>
<tr>
<td>Pedestrian density (d)</td>
<td>1.0</td>
<td>Section 4.4.2 - Case 2</td>
</tr>
</tbody>
</table>

The load is based on load case 2, effects of 1st harmonic, and for a pedestrian density of 1 pedestrian/m². The applied load is calculated from table 4.14:

\[
q_2 = 1.0 \cdot 280 \cos(2\pi f_n t) \cdot 1.85 \sqrt{1/(A \cdot d)} \cdot \psi \cdot \frac{w_{eff}}{w_d} \tag{5.5}
\]

\[
q_2 = 3.42 \cos(2\pi f_n t) \quad [N/m^2] \tag{5.6}
\]

The load \(q_2\) from Eq. 5.6 is applied in a vertical direction with the forcing frequency \(f_w = f_n\) as shown in figure 5.6.

![Figure 5.6: Direction and distribution of applied load in order to excite mode 2.](image)
CHAPTER 5. CASE STUDIES 5.1. MUNKTELL

Figure 5.7:
Visualization of vertical peak accelerations over entire footbridge.

Figure 5.8:
Acceleration response for a node in the middle of the central span, $a_{\text{max}} \approx 0.40 \text{m/s}^2$.

The response in figure 5.8 and 5.7 has the maximum peak acceleration $a_{\text{max}} \approx 0.40 \text{m/s}^2$. This satisfies both the limits recommended by Eurocode of $0.7 \text{m/s}^2$, see section 4.1.2, and comfort range 1 - Maximum comfort defined by Sétra, see section 3.4.1, table 4.8.
5.1. MUNKTELL

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Mode 3
The third modeshape is presented in figure 5.9 and the corresponding calculations in table 5.4. Mode 3 shows both vertical and longitudinal movement. Longitudinal modes are rarely an issue on footbridges and is therefore only treated as a vertical mode in this analysis [3].

Figure 5.9:
Third natural mode - Vertical/Longitudinal

Table 5.4: Munktell calculations - Mode 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency $f_n$</td>
<td>3.2606 Hz</td>
<td>Section 5.1.1 - Table 5.1</td>
</tr>
<tr>
<td>Natural freq. range</td>
<td>Range 3</td>
<td>Section 4.4.1 - Table 4.9</td>
</tr>
<tr>
<td>Load case</td>
<td>Case 3</td>
<td>Section 4.4.1 - Figure 4.21</td>
</tr>
<tr>
<td>Reduction factor $\psi$</td>
<td>0.826</td>
<td>Section 4.4.2 - Figure 4.24</td>
</tr>
<tr>
<td>Pedestrian density $d$</td>
<td>1.0</td>
<td>Section 4.4.2 - Case 3</td>
</tr>
</tbody>
</table>

The load is based on load case 3, effects of 2nd harmonics, and for a pedestrian of 1 pedestrian/m$^2$. The applied load is calculated from table 4.16:

$$q_{3,\text{vert}} = 1.0 \cdot 70 \cos(2\pi f_w t) \cdot 1.85 \sqrt{1/(A \cdot d)} \cdot \psi \cdot \frac{w_{eff}}{w_d}$$ (5.7)

$$q_{3,\text{vert}} = 5.702 \cos(2\pi f_w t) \quad [N/m^2]$$ (5.8)

The load $q_{3,\text{vert}}$ from Eq. 5.8 is applied in a vertical direction with the forcing frequency $f_w = f_n$ as shown in figure 5.10.

Figure 5.10:
Direction and distribution of applied load in order to excite mode 3 in the vertical direction.
The response in figure 5.12 and 5.11 has the maximum vertical peak acceleration $a_{\text{max}} \approx 0.12 \text{m/s}^2$. This satisfies both the limits recommended by Eurocode of $0.7 \text{m/s}^2$, see section 4.1.2, and comfort range 1 - Maximum comfort defined by Sétра, see section 4.4.1, table 4.8.
Mode 4
The fourth modeshape is presented in figure 5.13 and the corresponding calculations in table 5.5.

![Figure 5.13: Fourth natural mode - Vertical](image)

Table 5.5: Munktell calculations - Mode 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency $f_n$</td>
<td>4.04885 Hz</td>
<td>Section 5.1.1 - Table 5.1</td>
</tr>
<tr>
<td>Natural freq. range</td>
<td>Range 3</td>
<td>Section 4.4.1 - Table 4.9</td>
</tr>
<tr>
<td>Load case</td>
<td>Case 3</td>
<td>Section 4.4.1 - Figure 4.21</td>
</tr>
<tr>
<td>Reduction factor $\psi$</td>
<td>1.0</td>
<td>Section 4.4.2 - Figure 4.24</td>
</tr>
<tr>
<td>Pedestrian density $d$</td>
<td>1.0</td>
<td>Section 4.4.2 - Case 3</td>
</tr>
</tbody>
</table>

The load is based on load case 3, effects of 2nd harmonics, and for a pedestrian of 1 pedestrian/m². The applied load is calculated from table 4.16:

\[
q_4 = 1.0 \cdot 70 \cos(2\pi f_w t) \cdot 1.85 \sqrt{1/(A \cdot d)} \cdot \psi \cdot \frac{W_{eff}}{w_d} \quad (5.9)
\]

\[
q_4 = 6.905 \cos(2\pi f_w t) \quad [N/m^2] \quad (5.10)
\]

The load $q_4$ from Eq. 5.10 is applied in a vertical direction with the forcing frequency $f_w = f_n$ as shown in figure 5.14.

![Figure 5.14: Direction and distribution of applied load in order to excite mode 4.](image)
CHAPTER 5. CASE STUDIES

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Figure 5.15:
Visualization of vertical peak accelerations over entire footbridge.

Figure 5.16:
Acceleration response for a node in the centre of one of the the outer spans, $a_{\text{max}} \approx 0.61\text{m/s}^2$.

The response in figure 5.16 and 5.15 has the maximum peak acceleration $a_{\text{max}} \approx 0.61\text{m/s}^2$. This satisfies both the limits recommended by Eurocode of $0.7\text{m/s}^2$, see section 4.1.2, and comfort range 2 - Average comfort defined by Sétta, see section 4.4.1, table 4.8.
Mode 5

The last modeshape is presented in figure 5.17 and the corresponding calculations in table 5.6.

![Figure 5.17: Fifth natural mode - Vertical](image)

<table>
<thead>
<tr>
<th>Table 5.6: Munktell calculations - Mode 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Natural frequency ( f_n )</td>
</tr>
<tr>
<td>Natural freq. range</td>
</tr>
<tr>
<td>Load case</td>
</tr>
<tr>
<td>Reduction factor ( \psi )</td>
</tr>
<tr>
<td>Pedestrian density ( d )</td>
</tr>
</tbody>
</table>

The load is based on load case 3, effects of 2nd harmonics, and for a pedestrian of 1 pedestrian/m². The applied load is calculated from table 4.16:

\[
q_5 = 1.0 \cdot 70 \cos(2\pi f_w t) \cdot 1.85 \sqrt{1/(A \cdot d)} \cdot \psi \cdot \frac{w_{eff}}{w_d} \tag{5.11}
\]

\[
q_5 = 2.990 \cos(2\pi f_w t) \quad [N/m^2] \tag{5.12}
\]

The load \( q_5 \) from Eq. 5.12 is applied in a vertical direction with the forcing frequency \( f_w = f_n \) as shown in figure 5.18.

![Figure 5.18: Direction and distribution of applied load in order to excite mode 5.](image)
CHAPTER 5. CASE STUDIES

5.1. MUNKTELL

Figure 5.19:
Visualization of vertical peak accelerations over entire footbridge.

Figure 5.20:
Acceleration response for a node in the centre of one of the outer spans, \( a_{\text{max}} \approx 0.36 \text{m/s}^2 \).

The response in figure 5.20 and 5.19 has the maximum peak acceleration \( a_{\text{max}} \approx 0.36 \text{m/s}^2 \). This satisfies both the limits recommended by Eurocode of \( 0.7 \text{m/s}^2 \), see section 4.1.2, and comfort range 1 - Maximum comfort defined by Sétra, see section 4.4.1, table 4.8.
5.1.2 Sétra Method - Summary

The comfort requirements for case Munktell are satisfied regarding the recommendations set by Eurocode of $0.7m/s^2$ for vertical vibrations and $0.2m/s^2$ for lateral vibrations. The lowest comfort level as defined by Sétra occurred in the natural frequency $f_n \approx 4.05Hz$, 2nd harmonic of walking frequency $f_w = 2.025Hz$, where the comfort level was determined to be range 2 - Average comfort. A brief summary of the results is presented in table 5.7.

Table 5.7: Results from analysis with method from Sétra on case "Munktell"

<table>
<thead>
<tr>
<th>Mode</th>
<th>Harmonic</th>
<th>$\psi$</th>
<th>$q_{n,\text{amp}}$</th>
<th>$f_w[Hz]$</th>
<th>$f_{n1}[Hz]$</th>
<th>$M_{n1}[kg]$</th>
<th>$a_{\text{max}}[m/s^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2nd</td>
<td>1.000</td>
<td>0.690</td>
<td>2.0153</td>
<td>2.0153</td>
<td>173 853</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>1st</td>
<td>0.124</td>
<td>3.420</td>
<td>2.5381</td>
<td>2.5381</td>
<td>87 804</td>
<td>0.40</td>
</tr>
<tr>
<td>3</td>
<td>2nd</td>
<td>0.826</td>
<td>5.702</td>
<td>1.6303</td>
<td>3.2606</td>
<td>419 680</td>
<td>0.12</td>
</tr>
<tr>
<td>4</td>
<td>2nd</td>
<td>1.000</td>
<td>6.905</td>
<td>2.0243</td>
<td>4.0485</td>
<td>169 806</td>
<td>0.61</td>
</tr>
<tr>
<td>5</td>
<td>2nd</td>
<td>0.433</td>
<td>2.990</td>
<td>2.3268</td>
<td>4.6535</td>
<td>84 956</td>
<td>0.36</td>
</tr>
</tbody>
</table>

It is worth noting that the acceptable response for mode 2 depends completely on the reduction factor $\psi$. In order to ensure satisfying performance of the completed structure it might be reasonable to assume a situation where the natural frequency of mode 2 is closer to the natural walking frequency. This might occur if the mass is increased or the stiffness reduced. Assuming that the natural frequency is within the range 1.7 - 2.1 Hz the reduction factor becomes $\psi = 1.0$. This results in a higher dynamic load according to table 4.12. The dynamic load for mode 2 with the adjusted reduction factor becomes:

$$q_2 = 27.58 \cos(2\pi f_w t) \ [N/m^2]$$ (5.13)

This load is applied as previously in order to determine the maximum accelerations.

Figure 5.21:
Visualization of vertical peak accelerations over entire footbridge, $\psi = 1.0$.

Figure 5.22:
Acceleration response for a node in the middle of the central span, $a_{\text{max}} \approx 3.2m/s^2$
The maximum accelerations for mode 2 with the adjusted reduction factor results in accelerations 
> 3m/s², far more than what is acceptable even by the minimum comfort limit defined by Séta in 
table 4.8.

5.1.3 Generalized SDOF Method - Calculations

For a detailed description of the program developed for implementing the generalized SDOF method 
as described in section 4.3 the reader is referred to the manual in the appendix of this report. In order 
to acquire comparable results the parameters are initially set as described by Séta.

Vertical vibrations

The vertical modes are analysed for the forcing frequency range of 1.0 Hz - 2.8 Hz. The dynamic load 
factors are set as defined by Séta for vertical vibrations and the first two harmonics are included in the 
analysis. Furthermore, a frequency dependant reduction factor $\psi$ is applied according to the method by 
Séta.

The geometry data is inserted but the reduction factor $\rho$ described in section 4.3.1 is disabled in or-
der to acquire comparable results to section 5.1.1. The necessary modal data is read from the input file 
generated from the FE - model. Since the footbridge was assumed to be in traffic class I the applied 
load case is Very dense crowd. The scale factor is set to 0.915 to account for the effective width of the 
deck as described in previous section. See figure 5.23.

![Figure 5.23: Vertical input parameters for case Munktell](image)

Lateral vibrations

The lateral vibrations are treated in a similar manner as the vertical vibrations. Only the first mode is 
included and the dynamic load factors are set as described by Séta for lateral vibrations. See figure 
5.24.
**5.1. MUNKTELL**

**CHAPTER 5. CASE STUDIES**

**Figure 5.24:**
Lateral input parameters for case Munktell

**Results**
The results for vertical and lateral vibrations are illustrated in figure 5.25 - 5.28 as the maximum peak acceleration $a_{\text{peak}}$ and response factor $R$ by considering the maximum response in all nodes of the deck for a given forcing frequency.

**Figure 5.25:**
Vertical result spectrum as maximum peak accelerations $a_{\text{peak}} [m/s^2]$.

**Figure 5.26:**
Vertical result spectrum as response factor $R$. 

---

**Modal Data**

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Eigen Value</th>
<th>Mod. Shape No.</th>
<th>Mod. Shape Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>[3,1,32]</td>
</tr>
</tbody>
</table>

**Geometry**

<table>
<thead>
<tr>
<th>Area</th>
<th>Max Span Length</th>
<th>Deck Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>265</td>
<td>82</td>
<td>1.3</td>
</tr>
</tbody>
</table>

**Damping**

| Critical Damping Ratio | 0.08          |

---

**Loads**

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Load Type</th>
<th>Number of Loadcases</th>
<th>Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[3,1,32]</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Interpolation Values</th>
<th>Response Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>[3,1,32]</td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>[3,1,32]</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>[3,1,32]</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>[3,1,32]</td>
<td></td>
</tr>
</tbody>
</table>

---

**Frequency Reduction Factors**

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Interpolation Values</th>
<th>Reduction Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>[3,1,32]</td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>[3,1,32]</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>[3,1,32]</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>[3,1,32]</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.27: Lateral result spectrum as maximum peak accelerations $a_{\text{peak}}[m/s^2]$. 

Figure 5.28: Lateral result spectrum as response factor $R$

From figure 5.26 a maximum vertical response factor of $R \approx 78$ is found around 2 Hz, the second harmonic of the natural frequency of mode 4. This value exceeds the recommended value of 60, see section 4.3.1. The maximum lateral response factor in figure 5.28 is $R \approx 12$, however, the response factor is defined with regard to comfort requirements and do not account for lock-in effects, therefore the limit of $0.1m/s^2$ should not be exceeded regardless of response factor.

**Additional calculations**

As in section 5.1.1 it is of interest to analyse a situation where the natural frequency of the first vertical mode coincides with the natural walking frequency range $1.7 - 2.1Hz$, i.e the reduction factor $\psi$ is set to 1.0. This is achieved by disabling this option in the program.

Figure 5.29: Vertical peak acceleration spectrum for case Munktell, reduction factor disabled.

It is clear that the natural frequency at 2.54 Hz in figure 5.29 causes unacceptably large vertical accelerations. From the figure the peak acceleration is $a_{\text{max}} \approx 3.1m/s^2$. This is similar to the results of $3.2m/s^2$ for the same mode acquired through the Sétä method, see figure 5.22.
For comparison of load models the case is analysed once more using the method and parameters described by Willford & Young. The dynamic load factors used are frequency dependant based on measurements by S. Kerr and include the first four harmonics, see table 4.6. See figure 5.30 and 5.31 for DLF and load model. Furthermore, no frequency dependant reduction factor $\psi$ is applied this time, only the geometry dependant reduction factor $\rho$.

Since the first four harmonics are included in this analysis all modes with a natural frequency within the range $1.0 \text{ Hz} - 11.2 \text{ Hz}$ are included. The results are presented as a peak acceleration spectrum and a response factor spectrum in figure 5.32 and 5.33.

The results in figure 5.32 and 5.33 exceeds both the recommended values specified in Eurocode and the response factor limit of 60 defined by Willford & Young. The frequency dependant dynamic load factors used in this analysis increase with walking frequency, for natural frequencies in the upper range of $1.0 - 2.8 \text{ Hz}$ the load model exceeds the constant load factor of 0.4 used by Sétar. This combined with the absence of the frequency dependant reduction factor $\psi$ adopted in the method by Sétar results in increased vertical acceleration for the first vertical mode at $2.54 \text{ Hz}$. It is also clear that even though a much larger number of modes are included in the analysis due to inclusion of the four lowest harmonics, compared to previously only two, the main response is caused by the two lowest harmonics.
5.1.4 Generalized SDOF Method - Summary

The results are summarized in table 5.8 and 5.9 as maximum peak accelerations.

Table 5.8: Summary of vertical and lateral results on case Munktell, parameters by Sëtra.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f_n [Hz]$</th>
<th>Harmonic</th>
<th>$f_w [Hz]$</th>
<th>$a_{max} [m/s^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0153</td>
<td>2nd</td>
<td>2.0153</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>2.5381</td>
<td>1st</td>
<td>2.5381</td>
<td>0.39</td>
</tr>
<tr>
<td>3</td>
<td>3.2606</td>
<td>2nd</td>
<td>1.6303</td>
<td>0.12</td>
</tr>
<tr>
<td>4</td>
<td>4.0485</td>
<td>2nd</td>
<td>2.0243</td>
<td>0.59</td>
</tr>
<tr>
<td>5</td>
<td>4.6535</td>
<td>2nd</td>
<td>2.3268</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 5.9: Summary of vertical results on case Munktell, parameters by Willford & Young.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f_n [Hz]$</th>
<th>Harmonic</th>
<th>$f_w [Hz]$</th>
<th>$a_{max} [m/s^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.5381</td>
<td>1st</td>
<td>2.5381</td>
<td>4.20</td>
</tr>
<tr>
<td>2</td>
<td>2.5381</td>
<td>2nd</td>
<td>1.2691</td>
<td>0.94</td>
</tr>
<tr>
<td>4</td>
<td>4.0485</td>
<td>2nd</td>
<td>2.0243</td>
<td>0.42</td>
</tr>
<tr>
<td>5</td>
<td>4.6535</td>
<td>2nd</td>
<td>2.3268</td>
<td>0.59</td>
</tr>
<tr>
<td>6</td>
<td>5.2715</td>
<td>2nd</td>
<td>2.6358</td>
<td>0.93</td>
</tr>
</tbody>
</table>

The results in table 5.8 are presented in figures 5.34 to 5.39 as maximum peak acceleration distribution over footbridge deck.
5.1.5 Generalized SDOF Method - Hand Calculations

The results from section 5.1.3 and 5.1.4 are verified in table 5.10 using Eq. 4.35. As in section 5.1.3 the reduction factor \( \rho \) is not included when applying the parameters by Sétta.

\[
a_{\text{peak},m} = \frac{P_m}{M_m} \frac{1}{2\xi_m} \mu_{r,m} \tag{5.14}
\]

where the generalized force is determined through Eq. 2.53 based on a modeshape \( \phi_m \) and \( M_m \) is the generalized mass acquired from the FE software. Including the dynamic load factor and reduction factor \( \psi \) Eq. 5.14 becomes

\[
a_{\text{ver},m} = \frac{\text{DLF}_{h,m} \psi_{h,m} \hat{p}_m}{M_m} \frac{1}{2\xi_m} \mu_{r,m} \tag{5.15}
\]

See table 5.10 for a summary and comparison of results.

Table 5.10: Verification of peak accelerations using Eq. 5.15.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \xi_m )[%]</th>
<th>( M_m )[kg]</th>
<th>( \psi_{h,m} )</th>
<th>DLF_{h,m}</th>
<th>( \hat{p}_m ) [N]</th>
<th>( a_{\text{max},m} ) [m/s^2]</th>
<th>( a_{\text{ver},m} ) [m/s^2]</th>
<th>( a_{\text{ver},m} ) - ( a_{\text{max},m} ) [m/s^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Lat.</td>
<td>0.6</td>
<td>173 853</td>
<td>1.00</td>
<td>0.01</td>
<td>12 193</td>
<td>0.06 m/s^2</td>
<td>1.000</td>
<td>0.06 m/s^2</td>
</tr>
<tr>
<td>2 Vert.</td>
<td>0.6</td>
<td>87 804</td>
<td>0.12</td>
<td>0.40</td>
<td>8 122</td>
<td>0.39 m/s^2</td>
<td>1.000</td>
<td>0.38 m/s^2</td>
</tr>
<tr>
<td>3 Vert.</td>
<td>0.6</td>
<td>419 680</td>
<td>0.82</td>
<td>0.10</td>
<td>8 770</td>
<td>0.12 m/s^2</td>
<td>0.726</td>
<td>0.10 m/s^2</td>
</tr>
<tr>
<td>4 Vert.</td>
<td>0.6</td>
<td>169 806</td>
<td>1.00</td>
<td>0.10</td>
<td>11 360</td>
<td>0.59 m/s^2</td>
<td>1.000</td>
<td>0.56 m/s^2</td>
</tr>
<tr>
<td>5 Vert.</td>
<td>0.6</td>
<td>84 956</td>
<td>0.43</td>
<td>0.10</td>
<td>7 813</td>
<td>0.37 m/s^2</td>
<td>1.000</td>
<td>0.33 m/s^2</td>
</tr>
</tbody>
</table>

\[
\Delta_{h,m} = \frac{a_{\text{ver},m} - a_{\text{max},m}}{a_{\text{ver},m}} \tag{5.16}
\]

5.1.6 Comparison of methods

The results from the two methods based on parameters by Sétta applied to the case Munktell in section 5.1.1 (\( a_{\text{setra}} \)), 5.1.3 (\( a_{\text{script}} \)) and 5.1.5 (\( a_{\text{handcalc}} \)) are compared in table 5.11. The results based on parameters by Willford & Young are not included in the comparison.

Table 5.11: Comparison of maximum peak accelerations for case Munktell.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( a_{\text{setra}} ) [m/s^2]</th>
<th>( a_{\text{script}} ) [m/s^2]</th>
<th>( \Delta_{\text{script}} ) [%]</th>
<th>( a_{\text{handcalc}} ) [m/s^2]</th>
<th>( \Delta_{\text{handcalc}} ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.06 m/s^2</td>
<td>0.06 m/s^2</td>
<td>0.0 %</td>
<td>0.06 m/s^2</td>
<td>-0.0 %</td>
</tr>
<tr>
<td>2</td>
<td>0.40 m/s^2</td>
<td>0.39 m/s^2</td>
<td>-2.5 %</td>
<td>0.38 m/s^2</td>
<td>-5.0 %</td>
</tr>
<tr>
<td>3</td>
<td>0.12 m/s^2</td>
<td>0.12 m/s^2</td>
<td>0.0 %</td>
<td>0.10 m/s^2</td>
<td>-16.7 %</td>
</tr>
<tr>
<td>4</td>
<td>0.61 m/s^2</td>
<td>0.59 m/s^2</td>
<td>-3.3 %</td>
<td>0.56 m/s^2</td>
<td>-8.2 %</td>
</tr>
<tr>
<td>5</td>
<td>0.36 m/s^2</td>
<td>0.37 m/s^2</td>
<td>2.8 %</td>
<td>0.33 m/s^2</td>
<td>-7.8 %</td>
</tr>
</tbody>
</table>

\[
\Delta_{\text{script}} = \frac{a_{\text{script}} - a_{\text{setra}}}{a_{\text{setra}}} \quad \Delta_{\text{handcalc}} = \frac{a_{\text{handcalc}} - a_{\text{setra}}}{a_{\text{setra}}} \tag{5.17}
\]

The two methods used for the case Munktell based on parameters by Sétta provides similar results in terms of peak acceleration. The accelerations determined by hand through the use of Eq. 5.15 seem to provide comparable results.
By implementing the generalized SDOF method as described by Willford & Young with the corresponding load parameters the resulting accelerations exceeds all limits with regard to comfort requirements. This can be compared to the results of the analysis using parameters by Sétra where the limits of average comfort were satisfied. These results are discussed more later in this report.

Mode 3 is treated as a vertical mode, however, based on the maximum modal value for the vertical direction it seems to be dominated by longitudinal movement, see table 5.10.

## 5.2 Västberga

### 5.2.1 Sétra Method - Calculations

The bridge is assumed to be subjected to standard use with occasional crossing of large crowds, therefore traffic class III is assumed. However, this class does not require to consider the effects of 2nd harmonic and thereby no calculations are required since all natural frequencies are $\geq 2.6\,Hz$. For the purpose of this report the bridge class II is assumed. See section 1.3.2 for a description of the footbridge. As for the case Munktell the model is treated as it would for the ULS, i.e the same behaviour for small deformations as for large. This is considered a conservative assumption.

As for the case Munktell two situations are considered, an unloaded footbridge and a fully loaded footbridge. All modes with a natural frequency $< 5Hz$ are presented in table 5.12.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f_{n1}[Hz]$</th>
<th>$f_{n2}[Hz]$</th>
<th>$M_{n1}^b[kg]$</th>
<th>$M_{n2}^b[kg]$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.1837</td>
<td>3.1932</td>
<td>7474</td>
<td>7406</td>
<td>Arch bending</td>
</tr>
<tr>
<td>2</td>
<td>3.8917</td>
<td>4.0948</td>
<td>34529</td>
<td>31429</td>
<td>Vertical</td>
</tr>
<tr>
<td>3</td>
<td>4.5264</td>
<td>4.5489</td>
<td>9804</td>
<td>9676</td>
<td>Torsional</td>
</tr>
</tbody>
</table>

Since the fully loaded structure provides less favourable natural frequencies $f_{n1}$ compared to the unloaded structure $f_{n2}$ only the loaded structure is considered. The number of pedestrians is calculated based on the effective area of the deck and the pedestrian density.

$$A_{eff} = L \cdot w_{eff} = 40 \cdot 5 = 200 m^2$$

(5.18)

The total number of pedestrians N on the deck is defined as

$$N = A_{eff} \cdot d$$

(5.19)

The critical damping ratio is based on the material of the deck. For steel deck $\xi = 0.4\%$, see figure 2.9.

The first mode is bending of the arch, this is neither excited by pedestrian loads nor does this yield accelerations perceived by pedestrians, see figure 5.40. Therefore this mode is not included in the analysis.

![Figure 5.40: Mode1 - Mode 1 - First natural mode, Arch bending](image-url)
Mode 2

The first vertical mode is presented in figure 5.41 and the calculations in table 5.13.

![Second natural mode - Vertical](image)

Table 5.13: Västberga calculations - Mode 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency $f_n$</td>
<td>3.8917 Hz</td>
<td>Section 5.2.1 - Table 5.12</td>
</tr>
<tr>
<td>Natural freq. range</td>
<td>Range 3</td>
<td>Section 4.4.1 - Table 4.9</td>
</tr>
<tr>
<td>Load case</td>
<td>Case 3</td>
<td>Section 4.4.1 - Figure 4.21</td>
</tr>
<tr>
<td>Reduction factor $\psi$</td>
<td>1.0</td>
<td>Section 4.4.2 - Figure 4.24</td>
</tr>
<tr>
<td>Pedestrian density $d$</td>
<td>0.8</td>
<td>Section 4.4.2 - Case 3</td>
</tr>
</tbody>
</table>

The load is based on load case 3, effects of the 2nd harmonics, and for a pedestrian density of 0.8 pedestrian/m² based on traffic class II. The applied load is calculated from table 4.16.

$$q_2 = 0.8 \cdot 70 \cos(2\pi f_w t) \cdot 10.8 \sqrt{\xi/(A_{eff} \cdot d)} \cdot \psi$$  \hspace{1cm} (5.20)

$$q_2 = 3.024 \cos(2\pi f_w t) \quad [N/m^2]$$  \hspace{1cm} (5.21)

The load $q_2$ from Eq. 5.21 is applied in a vertical direction with the forcing frequency $f_w = f_n$ as shown in figure 5.42.

![Direction and distribution of applied load in order to excite mode 2.](image)
From figure 5.43 and 5.44 the maximum peak acceleration $a_{\text{max}} \approx 1.35m/s^2$ which exceeds the recommendations specified in Eurocode of $0.7m/s^2$, see section 3.1.2. However, it satisfies the minimum comfort requirements of $2.5m/s^2$ specified by Sëtra, see table 4.8.
Mode 3
The third natural mode is presented in figure 5.45 and calculations in table 5.14.

![Figure 5.45: Third natural mode - Torsional/Vertical](image)

Table 5.14: Västberga calculations - Mode 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency $f_n$</td>
<td>4.5264 Hz</td>
<td>Section 5.2.1 - Table 5.12</td>
</tr>
<tr>
<td>Natural freq. range</td>
<td>Range 3</td>
<td>Section 4.4.1 - Table 5.12</td>
</tr>
<tr>
<td>Load case</td>
<td>Case 3</td>
<td>Section 4.4.1 - Figure 4.21</td>
</tr>
<tr>
<td>Reduction factor $\psi$</td>
<td>0.592</td>
<td>Section 4.4.2 - Figure 4.24</td>
</tr>
<tr>
<td>Pedestrian density $d$</td>
<td>0.8</td>
<td>Section 4.4.2 - Case 3</td>
</tr>
</tbody>
</table>

The load is based on load case 3, effects of the 2nd harmonics, and for a pedestrian density of 0.8 pedestrian/m² based on traffic class II. The applied load is calculated from table 4.16.

$$q_3 = 0.8 \cdot 70 \cos(2\pi f_{w} t) \cdot 10.8 \sqrt{\frac{\xi}{(A_{\text{eff}} \cdot d) \cdot \psi}}$$  \hspace{1cm} (5.22)

$$q_3 = 1.790 \cos(2\pi f_{w} t) \quad [N/m^2]$$  \hspace{1cm} (5.23)

The load $q_3$ from Eq. 5.23 is applied in a vertical direction in order do excite the torsional mode with the forcing frequency $f_w = f_n$ as shown in figure 5.46 and 4.27.

![Figure 5.46: Direction and distribution of applied load in order to excite mode 3.](image)
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From figure 5.47 and 5.48 the maximum peak acceleration $a_{max} \approx 0.12m/s^2$ which satisfies both the recommendations specified in Eurocode of 0.7$m/s^2$, see section 3.1.2, and the maximum comfort requirements of 0.5$m/s^2$ specified by Sétra, see table 4.8.

5.2.2 Sétra Method - Summary

The comfort requirements for the case Västberga are not satisfied based the recommendations set by Eurocode of 0.7$m/s^2$ for vertical vibrations. However, it satisfies the lowest comfort class set by Sétra of 0.5$m/s^2$ for vertical accelerations. This is due to the large response computed for mode 2 caused by the 2nd harmonic of the walking frequency 1.9459$Hz$. A brief summary of the results is presented in table 5.15.

Table 5.15: Results from analysis with method from Sétra on case Västberga

<table>
<thead>
<tr>
<th>Mode</th>
<th>Harmonic</th>
<th>$\psi$</th>
<th>$q_{o,amp}$</th>
<th>$f_w[Hz]$</th>
<th>$f_{n1}[Hz]$</th>
<th>$M_{n1}[kg]$</th>
<th>$a_{max}[m/s^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2nd</td>
<td>1.000</td>
<td>3.024</td>
<td>1.9459</td>
<td>3.8917</td>
<td>34 529</td>
<td>1.35</td>
</tr>
<tr>
<td>3</td>
<td>2nd</td>
<td>0.592</td>
<td>1.790</td>
<td>2.2632</td>
<td>4.5264</td>
<td>9 804</td>
<td>0.12</td>
</tr>
</tbody>
</table>

5.2.3 Generalized SDOF Method - Calculations

For a detailed description of the program developed for implementing the generalized SDOF method the reader is referred to the manual in the appendix of this report. In order to acquire comparable results the parameters are set based on the method by Sétra. Since there are no lateral modes in the range of 0.5 – 2.5$Hz$ only the vertical modes are analysed.
Vertical vibrations
The input parameters are set in a similar manner to case Munktell, see section 5.1.3, and presented in figure 5.49.

Results
The results for vertical and lateral vibrations are illustrated in figure 5.50 and 5.51 as the maximum peak acceleration $a_{\text{peak}}$ and response factor $R$ by considering the maximum response in all nodes of the deck for a given forcing frequency.
5.2.4 Generalized SDOF Method - Summary

The results are summarized in table 5.16 as peak accelerations based on parameters by Sétra.

Table 5.16: Summary of vertical and lateral results on case Västberga.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( f_n ) [Hz]</th>
<th>Harmonic</th>
<th>( f_w ) [Hz]</th>
<th>( a_{\text{max}} ) [m/s^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.8917</td>
<td>2nd</td>
<td>1.9459</td>
<td>1.35</td>
</tr>
<tr>
<td>3</td>
<td>4.5264</td>
<td>2nd</td>
<td>2.2632</td>
<td>0.15</td>
</tr>
</tbody>
</table>

5.2.5 Generalized SDOF Method - Hand Calculations

The results from section 5.2.3 are verified in table 5.17 for each mode \( m \) using the equations and assumptions from section 5.1.5, see Eq. 5.15.

Table 5.17: Verification of peak accelerations for case Västberga

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \xi_m ) [%]</th>
<th>( M_m ) [kg]</th>
<th>( \psi_{h,m} )</th>
<th>( DLF_{h,m} )</th>
<th>( \dot{\beta}_{m} ) [N]</th>
<th>( a_{\text{max},h,m} )</th>
<th>( \mu_{\text{max},m} )</th>
<th>( a_{\text{ver},h,m} )</th>
<th>( \Delta_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Vert.</td>
<td>0.4</td>
<td>34 529</td>
<td>1.00</td>
<td>0.10</td>
<td>3729</td>
<td>1.35 m/s^2</td>
<td>1.000</td>
<td>1.35 m/s^2</td>
<td>0 %</td>
</tr>
<tr>
<td>3 Vert.</td>
<td>0.4</td>
<td>9 804</td>
<td>0.592</td>
<td>0.10</td>
<td>566</td>
<td>0.15 m/s^2</td>
<td>0.304</td>
<td>0.13 m/s^2</td>
<td>-13.3 %</td>
</tr>
</tbody>
</table>

where

\[
\Delta_h = \frac{a_{\text{ver},h,m} - a_{\text{max},h,m}}{a_{\text{ver},h,m}}
\]  
(5.24)

5.2.6 Comparison of methods

The results from the two methods based on parameters by Sétra applied to the case Västberga in section 5.2.1 (\( a_{\text{setra}} \)), 5.2.3 (\( a_{\text{adof,script}} \)) and 5.2.5 (\( a_{\text{adof,handcalc}} \)) are compared in table 5.18.

Table 5.18: Comparison of peak accelerations for each mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( a_{\text{setra}} )</th>
<th>( a_{\text{adof,script}} )</th>
<th>( \Delta_{\text{adof,script}} )</th>
<th>( a_{\text{adof,handcalc}} )</th>
<th>( \Delta_{\text{adof,handcalc}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.35 m/s^2</td>
<td>1.35 m/s^2</td>
<td>0 %</td>
<td>1.35 m/s^2</td>
<td>0 %</td>
</tr>
<tr>
<td>3</td>
<td>0.12 m/s^2</td>
<td>0.15 m/s^2</td>
<td>25 %</td>
<td>0.13 m/s^2</td>
<td>8.3 %</td>
</tr>
</tbody>
</table>

where

\[
\Delta_{\text{adof,script}} = \frac{a_{\text{adof,script}} - a_{\text{setra}}}{a_{\text{setra}}}
\]
\[
\Delta_{\text{adof,handcalc}} = \frac{a_{\text{adof,handcalc}} - a_{\text{setra}}}{a_{\text{setra}}}
\]  
(5.25)

The two methods used on the case Västberga based on parameters by Sétra provides identical results for the purely vertical mode in terms of peak accelerations. The third mode behaves in a torsional manner, the response determined by the Generalized SDOF method is slightly higher compared to the results computed with Abaqus. Furthermore the results determined through hand calculations provides fairly similar results.
5.2. VÄSTBERGA  

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Chapter 6

Discussion and Conclusions

6.1 Pedestrian load models

The pedestrian load model used throughout this report is the deterministic load model as presented in section 3.1. The parameters are based on measured footfalls where dynamic load factors aim to replicate an idealized footfall in a simple manner.

As seen for the case Munktell in figure 5.32 and 5.33 where the dynamic load factors by S. Kerr were used, the results are highly dependant on dynamic load factors and reduction factor. The dynamic load factor for the first harmonic from table 3.6 equals to 0.56 for walking frequencies above 2.5 Hz. This is compared to the corresponding DLF by Sétra, Bachmann or Schulze of \( \approx 0.4 \).

The longitudinal and lateral dynamic load factors in table 3.3 and 3.2 show visual similarities, see corresponding figures. However, they differ substantially on a fundamental level. The DLF by Schulze and Bachmann for both lateral and longitudinal loads of the first harmonic differ by a factor 2.5. This has clearly a considerable impact on the results acquired.

The varying models in terms of dynamic load factors are mainly due to the large variations in the human footfall and how these measurements have been performed. It has been mentioned that different load factors have been measured depending on the surface indicating that the dynamic load factor differ depending on the walking surface. A rigid surface might produce a higher initial impact from the foot while less rigid surface might dampen the impact. However, for the purpose of implementing as a design method it is reasonable to assume a situation which would generate the maximum dynamic load factors, in this case a rigid surface.

Crowd loading and vertical synchronization effects described in section 3.2 and 3.2.1 consider multiple pedestrians and interaction effects by introducing an equivalent number of pedestrians based statistical stimulations. The two situations considered are

- No vertical interaction - Normally distributed walking frequency centred around the natural frequency of the bridge and a random initial phase shift.
- Vertical interaction - Completely synchronized walking frequency but random initial phase shift.

This gave rise to two different relations describing the equivalent number of pedestrians on the footbridge, where for lightly damped structures the relation of number of equivalent pedestrians differ by a factor > 2 around the density 1 pedestrian per square meter, see figure 3.18. This simplification results in a very large increase in predicted load from a dense crowd to a very dense crowd, something that might be unlikely to happen in practice. However no conclusion can be drawn from this report, a comprehensive full scale experiment would be required.
6.2 Current Guidelines

In section 4.1 the current guidelines were described as presented in Eurocode and the document Bro 2004.

6.2.1 Eurocode

In EC0 comfort limits in terms of vertical and horizontal accelerations are presented, see table 4.1. These limits have been interpreted as Peak acceleration limits in this report.

For the case of vertical vibrations a limit of 0.7\( \text{m/s}^2 \) is recommended. This is compared to the limits specified by Sétro in table 4.8 where 0.5\( \text{m/s}^2 \) is defined as the limit between not perceived and merely perceived vibrations. Applying the limit recommended by Eurocode regardless of design situation might result in too conservative requirements for sparsely used footbridges. The approach by Sétro to define several comfort limits based on human perception increases the applicability since the limit should be based on predicted traffic and desired quality for the footbridge considered. Furthermore based on figure 2.15 the vertical peak acceleration limit for human perception for a walking frequency of 2\( \text{Hz} \) is \( a_{\text{baseline}} = 0.01 \text{m/s}^2 \). This corresponds to a response factor of 70 considering the recommendation in Eurocode, this is compared to the response factor 60 presented in section 4.3.1 by Willford & Young.

Considering horizontal vibrations a limit of 0.2\( \text{m/s}^2 \) is recommended for normal use and 0.4\( \text{m/s}^2 \) for exceptional use. However this does not account for horizontal synchronization effects, it seems to be based purely on perceived comfort by a pedestrian. Based on figure 2.14 and a vibration frequency of 1 − 2\( \text{Hz} \) the corresponding response factors are 39 for normal use and 78 for exceptional use. Based on the findings described by Sétro and Arup in section 3.2.2 it would seem that horizontal interaction effects might occur at accelerations much lower than the recommendations by Eurocode. By adopting the limits by Eurocode without considering horizontal synchronization the true accelerations will likely be substantially higher. This has been described as the reason for the large and unpredicted lateral vibrations observed during the opening of the Millennium bridge in 2000. Some further clarification in EC0 would be recommended in order to avoid lateral interaction effects.

As seen in section 5.1.3, where two different load models were applied to the case Munktell and acquiring completely different results, see table 5.8 and 5.9, the choice of load model parameters has a significant impact on the results. As no specific load model with corresponding parameters or even a pedestrian load is specified in EC0 the recommended limits are of no practical use in a design situation.

In EC1-2 it is recommended to perform a dynamic loading where the frequencies to consider are presented in table 4.2. For vertical vibrations during walking the recommended range is 1.0 − 3.0\( \text{Hz} \). This seems to be based on the first harmonic of the average walking frequency. However, it does not present higher harmonics of the walking frequency. Assuming the upper limit 3\( \text{Hz} \) as a walking frequency the 2nd harmonic is 6\( \text{Hz} \), something not mentioned in the document. As seen throughout section 5 the 2nd harmonic has a significant influence on the response and should not be disregarded.

6.2.2 Bro 2004

In the document Bro 2004 presented in section 4.1.1 a method to model dynamic loading is described. It defined a load based on footbridge geometry and natural frequency. No mention is made to either vertical or horizontal interaction effects and no physical interpretation of the force is made in terms of pedestrian density. It is therefore difficult to understand what the model is based on and what the corresponding load case is.
The model only includes natural frequencies within the range $0.0 - 3.5\,\text{Hz}$ and does not include harmonics of the walking frequencies as more recent load models do. Furthermore, this document is now replaced with the document TRVK Bro 11 which consists of references to Eurocode. Therefore this load model has not been included in the case studies in section 5, however, it is interesting to see that attempts have been made to present guidelines for examining the dynamic properties of a footbridge.

### 6.3 Generalized SDOF Method

The generalized SDOF method is described in section 4.3 and is based on a method published by the Concrete Center. The method as presented in this report was applied to the two cases in section 5 and compared to the method by Sétra where the response was computed using a FE-software.

#### 6.3.1 Implemented Program

The program developed for this report used to implement the generalized SDOF method is presented in the appendix as a manual. The scripts for the program are based on the theory presented throughout this report.

From table 5.11 for the case Munktell and table 5.18 for the case Västberga the scripted generalized SDOF approach seems to provide fairly similar response compared to the Sétra method where the response was computed using a FE-software. Based on the results it seems that the generalized - SDOF assumption is able to capture both purely vertical, lateral and longitudinal modes as well as torsional modes and combination modes. By only calculating the maximum steady state response the computational cost is reduced significantly.

A steady-state step is already implemented in Abaqus, however, the description is insufficient and the results have differed substantially from the results acquired through the use of Modal analysis. Therefore the steady-state step has not been used in this report.

The implemented program allows parameters such as natural frequency and generalized mass to be modified in a simple manner in order to study the sensitivity of the footbridge and how various parameters affect the response. For example, to study the effects of a reduced overall stiffness the designer can simply reduce the natural frequencies. The reduction in computational cost further allows the designer to study the effects of various load models and how these differ with regard to the response.

#### 6.3.2 Hand Calculations

Throughout section 5 the results obtained by the implemented program have been verified through simplified calculations applicable by hand. However, these calculations were performed for a single mode and a single forcing harmonic. As seen for both cases the results calculated by hand slightly underestimates the results from the implemented program and the computed results in Abaqus. This is because only the current mode is included in the analysis. It is true that the response for a forcing frequency that coincides with the natural frequency the response is dominated by the corresponding mode. However, as seen in Eq. 4.31, 4.32 and 2.39 the response from other modes generate a response as well, see figure 2.6. The response from these modes are substantially less compared to the current mode in resonance due to the term $f_n/f_m$. This is seen for the case Munktell where only one lateral mode was included in the analysis. The results calculated by hand seem to predict the response accurately due to the absence of other modes.

For preliminary calculations and verification of computed results it seems that the simplified hand calculations provides fairly accurate results.
6.4 Sétra method

6.4.1 Load Model

The load model used in the method is the deterministic load model as described in section 3.1 where the load is simulated using dynamic load factors and harmonics. The load factors used are within the range of other load models, see dynamic load factors by Bachmann in section 3.1, however, it is not clear how these load factors were established. If compared to the load factors by S. Kerr described in the method by Willford & Young, it is clear that the load factors differ significantly in the high and low end of the walking frequency range of 1.0 – 2.8 Hz. However, within the document by Sétra it is mentioned that the vertical dynamic load factor for the first harmonic of 0.4 should be increased to 0.5 for walking frequencies of 2.4 Hz and decreased to 0.1 for walking frequencies of 1.0 Hz. This suggests a frequency dependant load factor, however, this is only presented as a comment in the appendix and not included in the method.

In the document a frequency dependant reduction factors \( \psi \) was introduced to the load model to account for the likelihood of enough pedestrians walking with the same frequency as the natural frequency of the footbridge. This is a reasonable assumption, however, this requires the natural frequencies of the footbridge to be known. The predicted natural frequencies during the design stage might differ substantially to the natural frequencies of the completed footbridge because of various reasons presented in section 4.1. As seen in section 5.1.1 where the first vertical mode for the case Munktell had a natural frequency > 2.5 Hz. By including the reduction factor \( \psi \) the computed results satisfied the comfort requirements. However, by not including this reduction factor the results did not satisfy the comfort requirements. It is clear that the natural frequencies has a significant impact on the results using the method by Sétra and that some further studies regarding the predicted natural frequencies are required.

6.4.2 Modal analysis

The time stepping procedure usually implemented in FE-software are time consuming and as seen in section 4.2.5 prone to underestimate the results due to poor choice of time increments. Furthermore, determining the accelerations for the entire time period \( T \) is in this case unnecessary since only the maximum peak acceleration is of interest. By using equations such as Eq. 2.40 the maximum peak acceleration for the steady-state solution can be determined in a much more efficient manner. This would significantly reduce the computational cost of these types of problems.
CHAPTER 6. DISCUSSION AND CONCLUSIONS

6.5 CASE STUDIES

The two methods in this report have been applied to the cases Munktell and Västberga, see section 5. The results have been compared to recommendations in both Eurocode and Sétra.

6.5.1 Munktell

For the case Munktell where the method by Sétra was applied resulted in 0.61 m/s² as the maximum vertical peak acceleration and 0.06 m/s² as the maximum lateral peak acceleration. This satisfies both the requirements set by Eurocode and the average comfort range set by Sétra.

The comfort requirements for the modelled footbridge are clearly satisfied with regard to footfall induced vibrations. However, as seen in section 5.1.1 by reducing the lowest natural vertical frequency the resulting accelerations exceeds all requirements in terms of comfort. It is therefore of interest to early on measure the natural frequencies of the footbridge to ensure that the dynamic behaviour of the completed structure correspond to the model and thus ensuring satisfying comfort levels.

As mentioned throughout this report the behaviour of a footbridge during the ULS might differ substantially compared to the SLS. This is especially clear when considering the spring supports used to model the soil stiffness. As seen in table 4.5 where fixed supports yielded increased natural frequencies the boundary conditions have a significant influence of the overall stiffness of the footbridge. In section 5.1 the dynamic response was studied based on the assumption of spring supports based on TRVR Bro 11. This approach might result in an overestimation of the soil stiffness during dynamic loading due to the magnitude and time period of pedestrian loads. This results in increased natural frequencies of the completed structure compared to the FE-model and for this case considered beneficial. However, in order to account for the dynamic stiffness due to pedestrian loading in a more accurate manner more research on the subject would be required.

6.5.2 Västberga

For the case Munktell where the method by Sétra was applied resulted in the maximum vertical peak acceleration of 1.35 m/s². This does not satisfy the requirements set by Eurocode, however, as discussed in section 6.2.1, these requirements might be too strict depending on the expected usage of the footbridge. The minimum comfort requirements set by Sétra are satisfied corresponding to perceived but not intolerable vibrations.

As described in section 5.2.1, the predicted traffic corresponds to a footbridge class III due to low expected usage. For this report it was instead assumed to be class II in order to require calculations. It is clear that a lower expected traffic results in lower peak accelerations and therefore the measured response is predicted to be lower.
6.6 General Conclusions

The aim of a design method is to acquire clear results indicating whether a footbridge satisfies the comfort requirements with regard to footfall induced vibrations or not. As seen throughout this report the acquired results depend to a great extent on how the footbridge is analysed i.e the specific load model used, the method of determining the response and the limits used to evaluate the response in terms of perceived comfort. This in turn is based upon assumptions made on the human footfall, the pedestrian behaviour and the structural behaviour of the footbridge. Therefore the acquired results are no more accurate than the assumptions made by the designer.

It is important to keep in mind that all assumptions described in this report have been made conservatively resulting in a conservative outcome. Since the effects of footfall induced vibrations as described in this report are assumed to affect only the comfort of a pedestrian and not the structural integrity of the bridge, the consequence of unsatisfied requirements are a matter of economical loss. Vibrations are regularly reduced after construction using a variety of dampers such as the tuned mass damper, a device designed to absorb the vibrations of a structure at a certain frequency. It is therefore clear that a higher risk of exceedance is acceptable during the design of a footbridge with regard to footfall induced vibrations compared to ULS design where the consequence of failure is significantly higher.

Recalling the results in the case studies where the footbridges Munktell and Västberga were studied. Munktell did satisfy the average comfort requirements according to Sétra, however, it was also shown that a reduction in natural frequencies could result in intolerable accelerations. The central supports were models using spring supports in order to account for the soil stiffness. The stiffness of these springs were based on the initial settlements and in this report argued to be a severe underestimation of the dynamic soil stiffness. Therefore it seems likely that the natural frequencies of the completed structure will exceed the natural frequency of the model and thereby ensuring adequate comfort. For the case Västberga the excitable natural frequencies did coincide with the second harmonic of the natural walking frequency and thus a small increase or decrease in stiffness will not result in increased peak accelerations.

With all this in mind the results of the case studies are considered acceptable with regard to the comfort requirements as described by Sétra. Västberga is not satisfied with regard to the vertical accelerations requirements set by Eurocode, however, as discussed in this report these requirements are considered too conservative and are therefore disregarded.

Both methods used to determine the response provided very similar results. As mentioned previously the use of generalized systems require substantially less computational time while providing insight in the parameters governing the response. However, a FE-software is still used to acquire the modeshapes used in the generalized system. Therefore, in order to be applicable in a real design situation the implemented program would require some integration with an already established FE-software such as Abaqus.

To further understand the effects of the human footfall and crowd loading would require more studies on the subject. Through dynamic testing of existing structures and comparison against a predicted response the pedestrian load models can be refined. Reducing the uncertainties of a pedestrian load model allows for less conservative assumptions and thereby a more efficient design. It is also of interest to understand whether the risk of exceeding the comfort requirements corresponds reasonably to the consequence of an actual exceedance. This could also be understood further through more comprehensive dynamic testing of existing structures.
6.7 Future Work

A list of suggested future works

- Perform dynamic testing of the footbridge Munktell in order to study the natural frequencies. To compare the accelerations determined in the case studies a crowd test could be performed.

- Study the natural walking frequencies of pedestrians in different environments. The reduction factor introduced by Sétra suggested $1.7 - 2.1$ Hz to be the most likely walking frequency, however, it is of interest to study how the environment and the location of the footbridge could affect this.

- Perform studies on the soil stiffness during dynamic loading and how this could be modelled using FE-software.
Bibliography


Chapter 7

Appendix

7.1 Appendix A - Manual

Figure 7.1: Manual: Data Tab

Figure 7.2: Manual: Input Tab
7.1. APPENDIX A - MANUAL

CHAPTER 7. APPENDIX

Table 7.1: Explanation of data tab, see figure 7.1.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select Directory</td>
<td>Sets the working directory for temporary files and results</td>
</tr>
<tr>
<td>Select Model File</td>
<td>Sets the path to the current .odb file to be analysed. Model files supported are Abaqus and BRIGADE/plus.</td>
</tr>
<tr>
<td>Generate PreData</td>
<td>Opens the .odb file and extracts the models and parts in order to define input file. Model and part to include in analysis can be defined by the user.</td>
</tr>
<tr>
<td>Generate Data File</td>
<td>Based on the selected model and part the program generates a node set to include in the analysis and initiates a frequency step in selected program and writes necessary output data to a text file &quot;Report.txt&quot;.</td>
</tr>
<tr>
<td>Read Data File</td>
<td>The data file generated in previous step is read, the number of modes to include is defined by the user.</td>
</tr>
<tr>
<td>Visualize Object</td>
<td>The data read in previous step is visualized as modeshapes. The corresponding frequency, modal mass and maximum modal values are presented as well. The user can switch through the modes defined in previous step using the buttons &quot;Next Mode&quot; or &quot;Previous Mode&quot;.</td>
</tr>
<tr>
<td>Rotate Object</td>
<td>Switches the lateral and vertical axis of the object.</td>
</tr>
<tr>
<td>Clear Temp File</td>
<td>A working copy of the .odb file is saved in a temporary file in the working directory as well as other temporary files necessary for the script. This button clears the temporary directory.</td>
</tr>
<tr>
<td>Next Step</td>
<td>Saves the data for each lateral and vertical mode defined by the user.</td>
</tr>
</tbody>
</table>
Table 7.2: Explanation of Input tab, see figure 7.2.

**Define analysis**

Define analysis: The user selects either “Vertical Vibrations” or “Lateral Vibrations” to be included in the analysis. The frequency range is specified as a starting frequency and an ending frequency. The analysis works in finite frequency steps where the user defines the number of steps per Hz. The data is sent to the program with the “Write Step Data” button.

**Dynamic load factor**

Dynamic load factor: The user selects the dynamic load factors to be used in the analysis. The predefined load factors are "SETRA Vertical", "SETRA Lateral", "Young/Kerr Vertical" and "Batchmann Lateral". By selecting “Manual Input” the user can manually define the dynamic load factors. The user can visualize the dynamic load model for a forcing frequency with the ”Plot Harmonic Load” button, see figure 7.3. The chosen parameters are sent to the program using the "Write Dynamic load factors” button.

**Freq. red. factor**

Freq. red. factor: As defined by Sétra it is convenient to introduce a frequency dependant reduction factor \( \psi \). This is selected by the user by first defining the number of harmonics to apply the reduction factor on. For each harmonic a frequency dependant reduction factor is defined using interpolation points and interpolation values. This can be illustrated using the "Plot red Fac.” button, see figure 7.4.

**Geometry**

Geometry: The geometry of the footbridge is used to determine the applied load and optionally a reduction factor \( \rho \) as described by Willford & Young. The reduction factor can be disabled by the user with the "Disable geom. reduction factor" box.

**Damping**

Damping: The damping is manually defined by the user as a critical damping ratio \( \xi_m \). This can be applied to all modes using the "Apply to all modes” button.

**Modal Data**

Modal Data: The mode numbers to include was defined by the user in previous tab. The natural frequencies \( f_n \) and generalized mass \( M_m \) was extracted in the "Generate Data File” and is included based on the selected modes. The damping vector is generated by the user, see "Damping”. All modal data can be adjusted directly by the user before sending to the program using the “Write Modal Data” button.

**Loads**

Loads: The load is applied as a generalized force \( P_m \) for the mode \( m \), see Eq. 4.31 and 4.32. The load cases used are based on the method by Sétra where an equivalent number of pedestrians are used. The user can choose from the predefined cases "Sparse Crowd”, "Dense Crowd” and "Very Dense Crowd”. The data is based on the weight of a single pedestrian, set to 700 N, but is adjustable by the user. A scale factor is included in order to scale the load if necessary. The "Modal Load Factor” is a factor relating the total force \( q(t) \cdot A \) to the generalized force \( P_m \) based on the mode shape \( \phi_m \) for mode \( m \). The user sends the load data to the program using the "Write Load Data” button.
7.1. APPENDIX A - MANUAL

CHAPTER 7. APPENDIX

Figure 7.5: Manual: Response Spectrum Tab

Table 7.3: Explanation of Response Spectrum tab, see figure 7.5.

**Response:**
- **Response factor**: The response factor is defined as described by Willford & Young where a baseline curve serves as a reference value for human perception and the response factor is a multiple of this value. Depending on the choice of analysis, vertical or lateral, different baseline curves are used. See figure 2.15 and 2.16.

**Response:**
- **Peak Acceleration**: The peak acceleration is computed using the method described in section 4.3.

**Plot:**
- **Maximum nodal response**: This box plots only the response spectrum for the node where the maximum response occurs.

**Plot:**
- **Maximum nodal response per mode**: This box plots plots for each resonance frequency the node where maximum response occurs. This results in a response spectrum for each resonance frequency.

**Plot:**
- **Maximum object response**: This box plots plots the maximum response for each frequency step, regardless of node. This serves as the maximum response considering the entire object.
Table 7.4: Explanation of Response Visualization tab, see figure 7.6.

<table>
<thead>
<tr>
<th>Vis. Data</th>
<th>: Visualize data as colour plots of the peak acceleration distribution for current object and resonance frequency. The user can go through all resonance frequencies using the buttons &quot;Next Res.&quot; and &quot;Prev. Res.&quot;.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generate Report</td>
<td>: Generates a report where all data sent to the program is presented. The results in terms of peak accelerations and response factors is presented for each resonance frequency. The report is generated as a .txt file in the current working directory and also presented in the Report Tab. See appendix B for example of generated report file.</td>
</tr>
</tbody>
</table>
7.2 Appendix B - Report File

INPUT:
- Area: ............................... 295.0 m²
- Load Case: ........................... Very Dense Crowd
- Pedestrian density: .................... 1.0 pedestrian/m²
- Single pedestrian load: .............. 700.0 N
- Number of equivalent pedestrians: ... 31.37 pedestrians
- Scale factor: ........................... 1.0
- Modal Load Factor: .................... [0.3991, 0.4309, 0.5582]
- Point loads used in script: ........... [8876.92, 9584.23, 12415.68] N

SETTINGS:
- Direction: .............................. Vertical
- DLP from: ............................... KERR
- Frequency range: ...................... 1.0 - 2.0 Hz
- Number of steps: ..................... 180.0

REDUCTION FACTORS:
- Geom dep. Red. fac: .................... Disabled
- Freq. dep. Red. fac:
  - Interpolation Points:
    Harmonic: 1: [1.3, 1.7, 2.1, 2.6]
    Harmonic: 2: [2.6, 3.4, 4.2, 5.0]
    Harmonic: 3: No reduction factor
    Harmonic: 4: No reduction factor
  - Interpolation Values:
    Harmonic: 1: [0, 1, 1, 0]
    Harmonic: 2: [0, 1, 1, 0]
    Harmonic: 3: No reduction factor
    Harmonic: 4: No reduction factor

MODAL DATA:
- Number of modes: ..................... 4
- EigenValues: ........................... [2.53811, 3.26062, 4.04851] Hz
- Modal mass: ........................... [67803.7, 419688.0, 169906.0] kg
- Critical damping ratio: .............. [0.006, 0.006, 0.006]

HARMONIC DATA:
- Number of harmonics: .................. 2

Figure 7.7: Example of generated report file, page 1
RESULTS

Resonance Frequencies: 2.54, 1.27, 1.63, 2.02 Hz

Resonance Frequency: 2.54 Hz
- Harmonic: 1
- Natural Frequency: 2.54 Hz
- Maximum Peak Acceleration: 0.5623 m/s^2
  - Node Index: 34
- Maximum Response Factor: 63.5538
  - Node Index: 34

Resonance Frequency: 1.27 Hz
- Harmonic: 2
- Natural Frequency: 2.54 Hz
- Maximum Peak Acceleration: 0.0 m/s^2
  - Node Index: 0
- Maximum Response Factor: 0.0
  - Node Index: 0

Resonance Frequency: 1.63 Hz
- Harmonic: 2
- Natural Frequency: 3.26 Hz
- Maximum Peak Acceleration: 0.6789 m/s^2
  - Node Index: 630
- Maximum Response Factor: 9.386
  - Node Index: 630

Resonance Frequency: 2.02 Hz
- Harmonic: 2
- Natural Frequency: 4.04 Hz
- Maximum Peak Acceleration: 0.5455 m/s^2
  - Node Index: 631
- Maximum Response Factor: 73.8763
  - Node Index: 166

--- RESPONSE FACTOR EXCEEDS 60 FOR RESONANCE FREQUENCY 2.54 Hz ---
--- RESPONSE FACTOR EXCEEDS 60 FOR RESONANCE FREQUENCY 2.02 Hz ---

Figure 7.8: Example of generated report file, page 2
7.3 Appendix C - Verification of Scripts

To verify the program for implementing the method as presented by Willford & Young the worked example in section 5.1 of the document “A design guide for footfall induced vibrations” is analysed [2]. The properties and results of the footbridge used in the worked example are summarized in figure 7.9.

The modal mass \( \hat{m} \) is estimated in the report using hand calculations and is approximately the same in all modeshapes, see Eq. 2.46 and 2.47 of this report.

\[
\hat{m} = 2 \cdot \frac{L \cdot m_L}{2}
\]

where \( m_L \) is the mass per unit length based on the approximate cross sectional area 0.77 \( m^2 \) and \( L \) is the length of one span.

\[
\hat{m} = 2 \cdot \frac{20 \cdot 1848}{2} = 36960 \text{ kg}
\]

The eigenfrequencies used in this analysis are measured frequencies presented by Willford & Young as 4.65 Hz, 6.56 Hz and 14.66 Hz for the three lowest vertical modes. The results are presented as a response factor for one pedestrian walking in the frequency range of 1.0 - 2.8 Hz, see figure 7.10.

The results for a node in midspan for the worked example calculated in the report by Willford & Young.
The example is now calculated using the scripts created for implementing this method. The footbridge is modelled in the FE-software BRIGADE/plus based on the sketch provided in figure 7.9. See figure 7.11.

![Figure 7.11](image)

Cross sectional sketch used in BRIGADE/plus.

The cross section is extruded and boundary conditions are set in such a way that modeshapes similar to the approximated ones are acquired. Using a frequency analysis the following vertical modeshapes corresponding to the approximated shapes in figure 7.9 are computed. See figure 7.12.

![Figure 7.12](image)

The three lowest vertical modeshapes computed using BRIGADE/plus.

The necessary data for this method is extracted from the FE-model using a Python script. In order to acquire comparable results the approximated values such as modal mass, eigenfrequency and critical damping ratio used in the worked example by Willford & Young are applied to this model as well. The result from the program is presented in figure 7.13.
Figure 7.13:
Response factor in midspan for the frequency range of 1.0 - 2.8 Hz calculated using scripts based on the method by Willford & Young.

Based on the comparison with the results from Willford & Young in figure 7.10 and the results from the script in figure 7.13 the implemented program used is determined to be valid.
7.4 Appendix D - Construction of Munktell

Photos of Munktell during construction. Photos by Ermias Bsrat at WSP Bro och Vattenbyggnad.

Figure 7.14:
Munktell - Overview 1

Figure 7.15:
Munktell - Overview 2

Figure 7.16: Munktell - Arch and column connection to support

Figure 7.17: Munktell - Arch and column connection to support